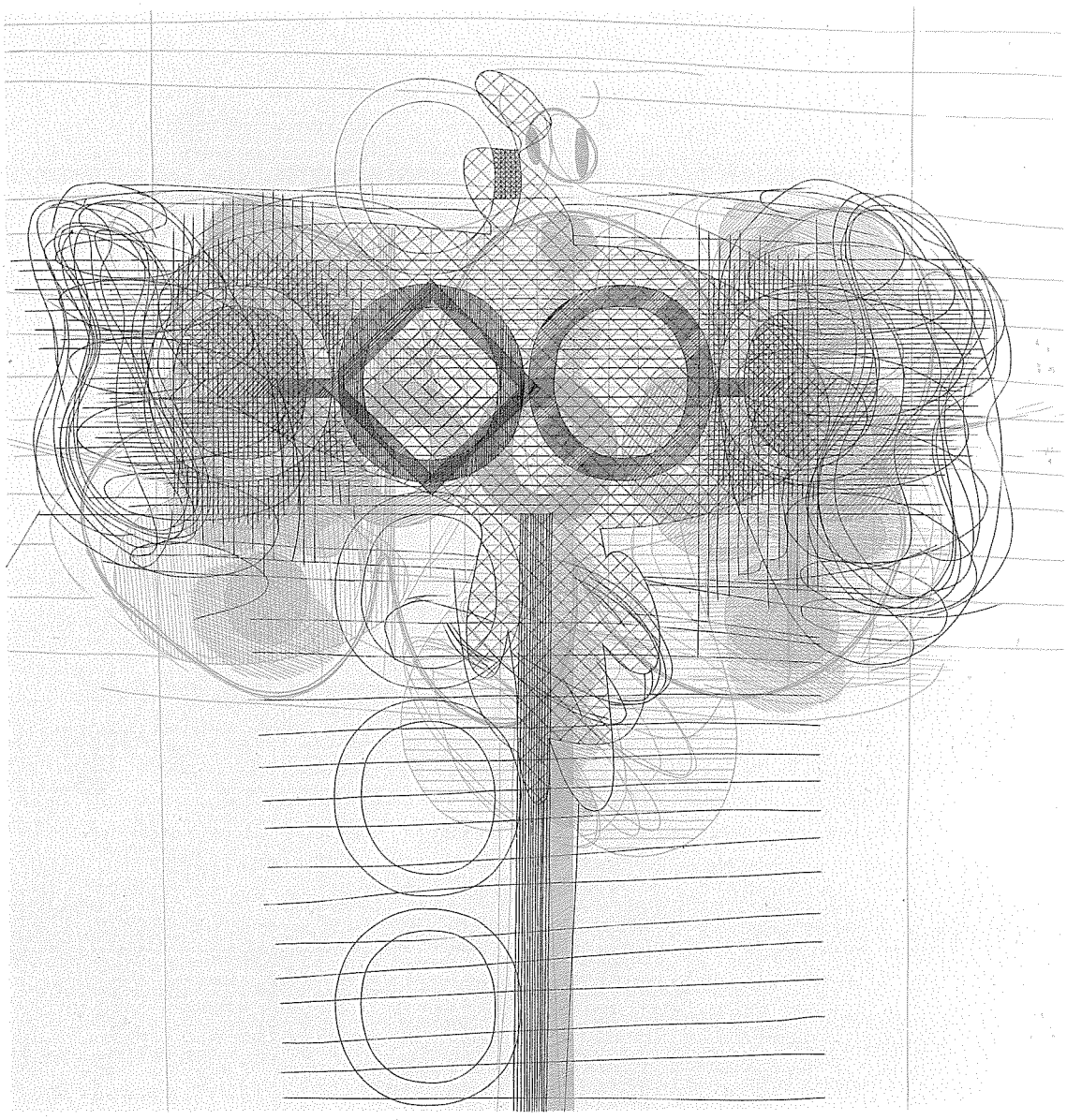
bernard schmitt

macroeconomic theory

a fundamental revision



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MACROECONOMIC THEORY
A FUNDAMENTAL REVISION

BERNARD SCHMITT

MACROECONOMIC THEORY

A FUNDAMENTAL REVISION

Translated from the French manuscript
by Sister Benvenuta Bras

EDITIONS CASTELLA
ALBEUVE · SWITZERLAND

PREFACE

This small book is the modest outcome of over ten years of research. Its main ideas have already been developed in La formation du pouvoir d'achat *(Sirey, Paris, 1960),* Monnaie, salaires et profits *(Presses Universitaires de France, Paris, 1966) and* L'analyse macroéconomique des revenus *(Dalloz, Paris, 1971). The object of persuading economists to reexamine critically the whole corpus of modern macroeconomics rests upon my long-established conviction that the soi-disant post-Keynesian or neo-Keynesian school is built around a faulty nucleus. The mistake is not to be found in the assumptions made about the real world. Instead, the dominant theory is accused of a much more serious defect. The theory of national income determination as it is generally expounded today depends on a basic logical error. Such faults in logic are easily ascertained provided they are clearly set forth.*

Sister BENVENUTA BRAS *translated the manuscript from French and Miss* ISOBEL GAWLER *made a number of useful suggestions which have been incorporated in the final text. The index has been compiled by* G. SOCCHI, *research assistant at Fribourg University.*

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CHAPTER I

PRICE AND INCOME DETERMINATION: A DECEPTIVE ANALOGY

A. POSITIVE ANALYSIS

1. Analogy, although a somewhat unreliable tool, can nevertheless be used to determine the equilibrium level of Y, the national income.

2. By definition, Y equals $C+I$. National income is the sum of the purchases, or demands, of consumer goods, C, and of investments, I.

3. The statement $Y=C+I$ equates total supply Y with total demand $C+I$. It follows that Y equals $C+I$ when and only when equilibrium is attained. For any other value of Y the equation will not hold. Cannot Y_0 be defined like the price of any commodity which equalizes supply and demand?

THEORY OF DETERMINATION OF NATIONAL INCOME VERSUS THEORY OF PRICE DETERMINATION IN DYNAMICS

4. Suppose a one-commodity world. The price of the goods is measured by a unit of account.

5. We should distinguish between the price of the commodity and the level of this price. Price is synonymous with exchange. Thus, the money price denotes the exchange between a certain quantity of the commodity, measured in physical units like pounds, yards,... and a certain sum of money, a given number of dollars, for example. The price level is the number of units which must be spent to obtain a physical unit of the merchandise. The price level is not a constant in time. If we have a supplier and a purchaser, price level, at any instant t_n, is determined when p equals p_n, p_n being the price level which at that instant equalizes supply and demand of the commodity.

At instants t_0, t_1, t_2, t_3, exchanges take place between money and the commodity. It is easy to determine the price level which obtains at

these successive, if perhaps irregularly spaced, instants. Thus, to find p_0, divide the total sum of money given for the commodity at t_0 by the number of physical units actually exchanged. If, at t_0, \$30 are paid for the acquisition of three pounds of the commodity, the price and its level are indicated in the two following expressions.

$$\text{Price: } \$30 = 3 \text{ lbs of the commodity}$$

$$\text{Price level: } \frac{\$30}{3 \text{ lbs}} = \$10/\text{physical unit}^1$$

6. One assumes that no purchase occurs in the intervals between instants t_0, t_1, t_2, t_3. Nevertheless, it is conceivable that all points representing price levels on the chart belong to a mathematically defined curve.

This continuous curve includes two categories of points. *Between* points $A, B, C, D, \ldots$, corresponding to exchanges which in fact take place at instants t_0, t_1, t_2, $t_3, \ldots$, other points on the curve indicate only the *level which price would reach at any instant t_n if an exchange were to take place at that instant.*

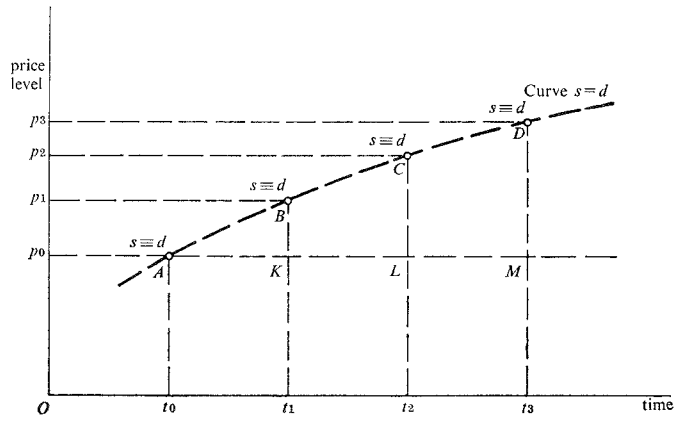


Fig. 1

All points on the curve correspond to the equality between supply and demand of the commodity, $s=d$. Yet, corresponding to instants t_0, t_1, t_2, $t_3, \ldots$, for points A, B, C, D, supply and demand are not merely equal but they coincide, $s \equiv d$.

The equality sign

$$s=d$$

represents the equality between supply and demand, when these two factors do not coincide in a single transaction, but remain separate.

The identity sign

$$s \equiv d$$

denotes more, because then the factors of supply and demand are compounded in a single operation, namely the exchange which takes place. At instants t_0, t_1, t_2, t_3,..., exchanges occur, so that *the same transaction* can be called either supply or demand according to the point of reference.

At any instant other than $t_0, t_1, t_2, t_3, \ldots$, no transaction can be observed, since no transaction takes place. *Supply and demand, two distinct factors, coincide only at A, B, C, D.*

7. The result of the preceding paragraph can be interpreted by dividing the time axis: (i) At instants t_1, t_2, t_3,..., the forces of supply and demand are merged into a single, double-sided transaction. (ii) At any other instant, on the contrary, supply and demand remain distinct factors.

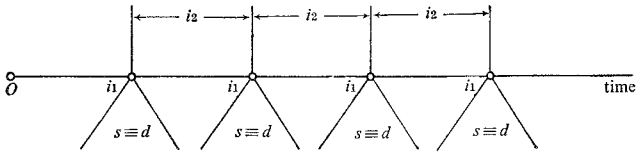


Fig. 2

For the sake of clarity, instants can be divided into two families.

Family of instants i_1.

Instants like t_0, t_1, t_2, t_3,..., when purchases take place belong to family i_1.

Family of instants i_2.

This family includes all other instants of real time. Any instant included in the time interval that elapses between two purchases, like time intervals $t_0 - t_1$, $t_1 - t_2$, $t_2 - t_3$, belongs to family i_2.

The number of instants i_1 is finite, while that of instants i_2 is infinite. There exists an infinity of instants when supply and demand remain separate, unequal or equal, but not identical.

8. At all instants i_2, infinite in number, supply and demand are distinct factors. At these moments, then, the commodity may be demanded in excess of its supply, or supplied in excess of its demand.

9. At any instant of family i_1, that is at the exact moment of any actual purchase, excess demand is a meaningless expression, since supply and demand form a single transaction.

10. Excess demand must be written

$$d \not\equiv s$$

and not

$$d \neq s.$$

At instants of family i_2, that is at any instant when no exchange occurs, excess demand is positive or negative. But excess demand can also be zero. The condition $d \not\equiv s$, in fact, offers three possibilities.

$$d \not\equiv s \begin{cases} d > s \\ d < s \\ d = s \end{cases}$$

Excess demand is zero when we can write simultaneously $d \not\equiv s$ and $d = s$. This is in no way contradictory since two separate factors can be equal to each other.

11. How are we to measure excess demand? Since we are reasoning in dynamics, excess demand should also be defined in time, even though usually excess demand is studied in static analysis. It is easy to show *dynamic excess demand* on the chart.

1° A base must be chosen, for instance price level p_0 as it occurs at t_0.

2° Excess demand to be measured must be specified at t_1, for example, or at any instant t_n.

3° At the instant chosen, the price level which equalizes supply and demand of the commodity must be known.

4° A unit of measure must be agreed upon. This unit is naturally the one used on the vertical axis to measure price level. Excess demand is equal to the variation of price level which is just sufficient to equalize supply and demand of the commodity. For normal goods, excess demand is positive if the price level is to increase. If it is to decrease, excess demand is negative.

All these conditions are represented on the graph by segments *KB*, *LC*, and *MD*.

—KB, measurable on the vertical axis, is the difference between level p_1 and level p_0 of the price of the commodity. Excess demand is positive. At t_1, an excess demand equal to KB would have built up if the price level had remained constant at level p_0.

—Segment LC represents positive excess demand at t_2. This means, clearly, that if the price level had remained until t_2 the same as at t_0, excess demand would be measured at t_2 by distance LC on the vertical axis.

—The same reasoning can be applied to any time point where purchases occur.

To sum up:

It is important to consider the two cases separately, according to whether curve $s=d$ is broken or not.

If curve $s=d$ is continuous, on base p_0 dynamic excess demand is known for every instant subsequent to t_0. On the graph, we see that it increases regularly as time elapses.

It is more likely that curve $s=d$ cannot be continuous. In this case, the only points known on the curve are A, B, C, D. Must we consequently infer that excess demand appears instantaneously at t_1, t_2, t_3,...? Certainly not. At these instants, no excess demand is definable, for condition $d \not\equiv s$ is lacking. Therefore it would be wrong to talk about excess demands at instants t_1, t_2, t_3,... *although the price level may be known only then.* The correct inference is simply that excess demand cannot be measured during time intervals $t_0 - t_1$, $t_1 - t_2$,..., etc., since we assume that curve $s=d$ is not continuous. But it does not follow from this that excess demand appears simultaneously with actual purchases, which would be absurd. Thus, excess demand measured at t_1 has its origin in the open interval from t_0 to t_1, appearing in real time at instants belonging to family i_2.

Excess demand is produced *only in the intervals of time elapsing between two successive purchases. Only instants belonging to family i_2 can give rise to excess demand.* If no interval of time passed between successive transactions, no excess demand could be defined, with the result that *variation of price level in time could no longer be explained.*

12. Figure 3 below summarizes the division of real time.

	Instants i_1
successive purchases	excess demand not conceivable $d \equiv s$

Instants i_2

intervals of time between successive purchases { excess demand conceivable positive, negative, or zero, $s \neq d$

Note again that at instants i_1 excess demand is not equal to zero: it simply does not exist whereas the number zero does exist. Excess demand can only be encountered at instants i_2.

Dynamic analysis of price variation is built entirely upon instants i_2.

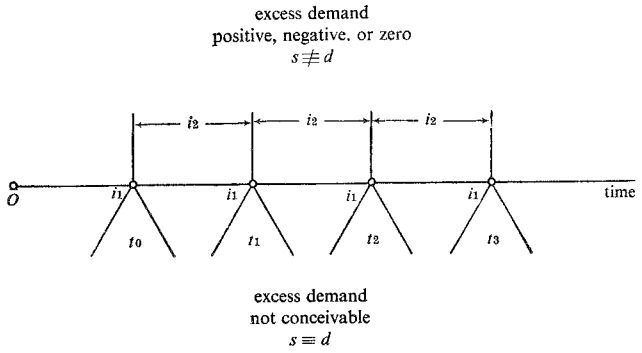


Fig. 3

Supply and demand of a real commodity are either distinct factors (instants i_2), or combined factors (instants i_1).

In the case where $d \neq s$, supply and demand can be weighed separately on a balance. If supply is greater than demand, $s > d$, or if demand is greater than supply, $d > s$, variation of price level will finally restore equilibrium.

In the case where $d \equiv s$, we cannot weigh demand on one side and supply on the other, since the two factors are inseparable. Identity $d \equiv s$ denotes a single transaction as seen from the seller's and the buyer's viewpoints. Neither is equality $s = d$ a satisfactory definition of the relationship between supply and demand at instants i_1. All equality results either from an equalization or from logical necessity. When $d \equiv s$, we have a logical or definitional identity. Supply cannot adapt to demand, nor demand to supply, since they are the two sides of the same transaction.

13. The assumed analogy would require dynamic determination of national income level to develop in the same way as dynamic determination of price level.

14. In price theory, we have the terms supply, demand, and price level of a single commodity or of a collection of commodities. What are the corresponding terms in income theory?

—When we pass from price to income theory, we obviously replace price level by the level of national income. The level of national income can be assessed by the number of workers measured in wage units per unit of time.

—In the field of income theory, *total demand* corresponds to the demand of a commodity in the field of price theory. Total demand is made up of the sum of all consumption and investment purchases, $D=C+I$.

—Similarly, *total supply* is equal to Y, the value of consumption and investment goods.

15. If the analogy between the theories of price and income determination is correct, then we may reason as follows. As time elapses, one of the two factors of total supply and total demand outgrows the other, so that the level of national income must vary until Y again equals $C+I$.

The chart of paragraph 6 can be read so as to define the level of *dynamic excess demand in national income analysis*. The level of national income is measured at instants t_0, t_1, t_2, t_3,..., and by simple observation is found to be equal to levels A, B, C, D on the vertical axis. Segments KB, LC, and MD measure excess of total demand over total supply: KB is the level of excess demand which would obtain at t_1 if national income were, at t_1, again at the same level as it was previously at t_0. Similarly an excess demand equal to LC would have emerged by instant t_2 if the level of national income had remained constant from t_0 to t_2. Excess demands KB, LC, and MD are positive and they explain the positive growth of national income in time. Thus, to eradicate excess demand MD at t_3, national income must be raised at that moment to level D on the vertical axis.

BREAKDOWN OF THE ASSUMED ANALOGY; $Y \equiv C+I$ AT EVERY INSTANT IN REAL TIME

16. The distinction between families of instants i_1 and i_2, as explained in paragraph 7, provides us with a useful tool of analysis.

Although in dynamic analysis of price level determination an infinite number of instants i_2 exists, these instants cannot exist in dynamic analysis of national income determination.

Excess demand, conceivable in dynamic price analysis, loses all meaning in national income analysis.

CONTINUOUS ANALYSIS

17. National income level and price level are both assumed to be known continuously in real time. At any instant after t_0, it is possible to determine (i) the price level which equalizes the supply and demand of the commodity and (ii) the national income level which adjusts total supply, Y, to total demand, $C+I$. A comparative graph (Fig. 4 and Fig. 5) discriminates on the time axis between the instants of family i_1 and the instants of family i_2.

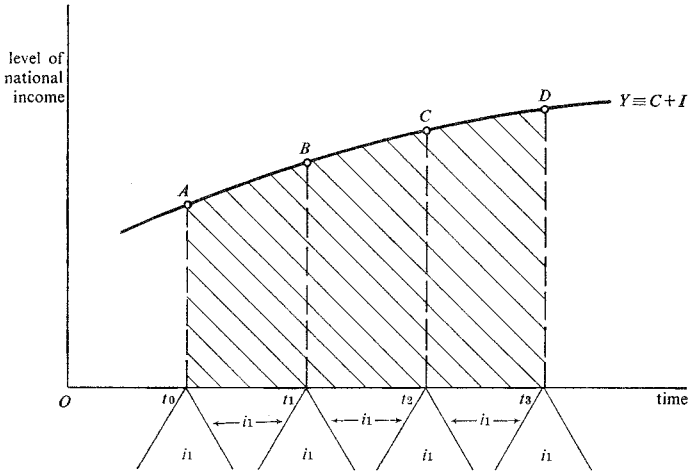


Fig. 4

On the two graphs, the results are diametrically opposed.

—As to price level, all points of equality between supply and demand lie on the curve showing the price level as a continuous function of time. Only a finite number of points on this curve correspond to the identity of factors, $s \equiv d$. All other points on the curve represent the result of adjustment between the two separate factors. In all intervals between successive purchases, supply and demand are in conflict, causing continuous variation of price level in time.

—With reference to national income level, the analysis retains only instants i_1. At any moment, factors Y and $C+I$ are identical, like the

two sides of the same coin. Points on the curve are no longer the result of adjustment between supply and demand, for they are *always identical*, $Y \equiv C+I$.

The diverging results may be attributed to a single fact. It requires a certain length of time to produce national income. In other words, no amount of national income can ever be produced instantaneously. It takes a whole day to produce the income of that day. But the realization of any transaction or price only takes an instant.

Take a finite interval of time, $t_0 - t_n$.

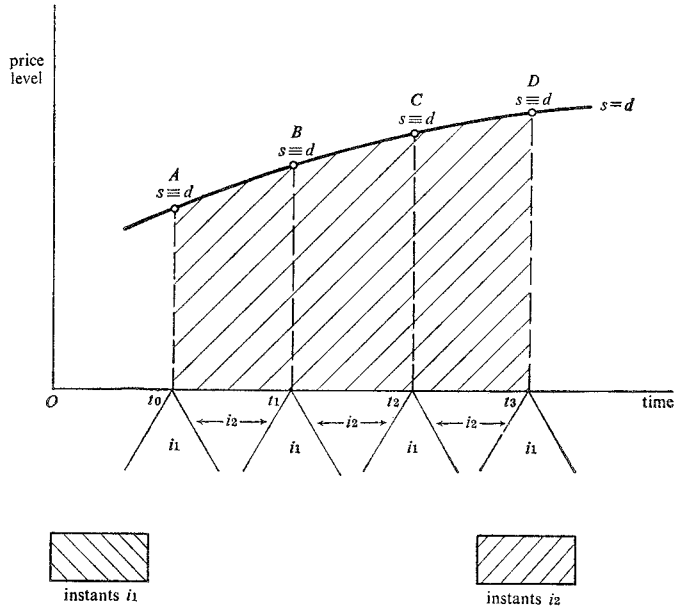


Fig. 5

—Is price measurable when t_n and t_0 coincide? At t_0, the price level is indeed known. The price realized at instant t_0 is p_0q_0, where q_0 is the quantity of the goods purchased.

—Similarly, is income measurable when t_n coincides with t_0? Now the answer is negative. It remains true, of course, that the level of income is measured at instant t_0 by vertical distance OA. But a finite time interval is necessary during which production continues. When interval $t_0 - t_n$ is reduced to zero, national income produced during that time must also be zero. Suppose three people are at work at instant t_0. This means that the level of national income is positive at t_0.

Yet national income produced at t_0 is nil, for output can only be positive if the three men stay at work for more than an instant. In other words, national income as against the level of national income can only be measured during a *finite* interval of time.

The following points summarize the argument.

I. *Hypothesis common to price and income theories.*

The price level and the level of national income are known for any instant in real time after t_0.

II. *Theorem common to price and income theories.*

Prices or transactions as realized and realized incomes imply identity of supply and demand.

$$\text{Realized price:} \quad s \equiv d$$

$$\text{Realized income:} \quad Y \equiv C + I$$

III. *Basic difference between price and income theories.*

Any realized transaction or price occurs instantaneously, whereas income is produced in a finite period of time.

IV. *Consequence of this basic difference.*

Price analysis

1. The price level being known at instants t_0, t_1, t_2, t_3,..., price is itself known at these instants. To ascertain price, we multiply the known levels p_0, p_1, p_2, p_3,..., by the corresponding physical quantities q_0, q_1, q_2, q_3, actually purchased at instants t_0, t_1, t_2, t_3,... The solution is determined, for quantities q_0, q_1, q_2, q_3,..., are instantaneously known at t_0, t_1, t_2, t_3.

2. Transactions are realized at the two instants t_0 and t_1, but none occurs during the interval between.

Income analysis

1. The level of income is known at instants t_0, t_1, t_2, t_3,... Yet it is impossible to determine the amount of national income produced at those instants. A finite period of time must elapse. The level must be multiplied by the time factor. Level of income × time = income produced. If the time factor is reduced to a zero interval, income produced is itself reduced to zero.

2. Income pertaining to period $t_0 - t_1$ is produced not only at t_0 and t_1 but also during the whole period between.

In short,

—*Price theory.* Identity of supply and demand is realized at t_0 and t_1, but not during the time period between, since prices are only realized

at t_0 and t_1, and not during the period between. Instants t_0 and t_1 belong to family i_1, and all other instants from t_0 to t_1 belong to family i_2.

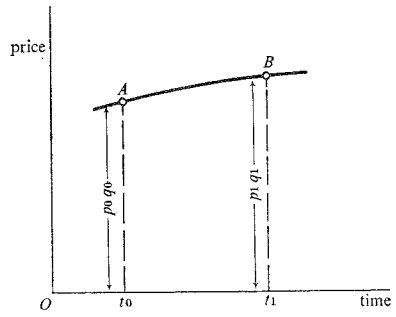


Fig. 6

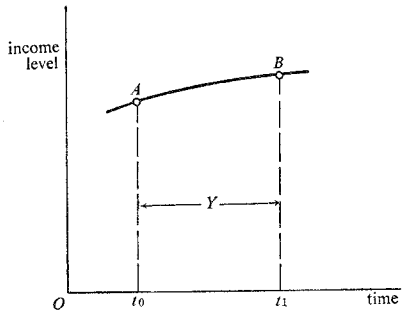


Fig. 7

—*Income theory.* Identity of total supply and demand is realized at t_0 and t_1. However, identity $Y=C+I$ extends further to occupy the whole time period between these instants. Income is being produced during the whole interval $t_0 - t_1$, so that it covers the entire period. Consequently, all instants from t_0 to t_1 belong to family i_1. Generally speaking, no matter what period of time is considered, all instants of that period, including the beginning and the end, belong to family i_1. This brings us to a conclusion with far-reaching results. Since in national income analysis it is logically impossible to find any instants of family i_2, dynamic excess demand is meaningless in this context.

18. The following graphs may further clarify the issue.

1. Determination of price level

Consider two transactions only which take place at t_0 and t_1.

In order to measure dynamic excess demand, we arbitrarily choose price level p_0 as realized at t_0. If this level were to continue up to t_n, between t_0 and t_1, a certain positive amount of excess demand equal to JS would appear during interval $t_0 - t_n$. As a result, price level p_0 would not balance supply and demand of the goods at t_n. In order to restore equilibrium, price level must rise at t_n from p_0 to p_n.

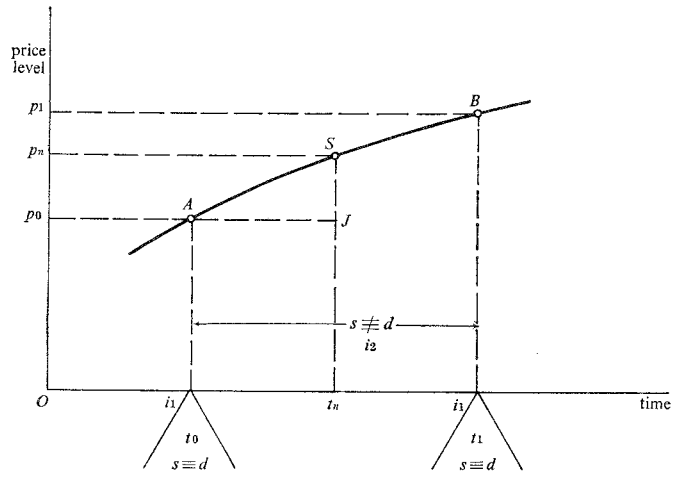


Fig. 8

Determination of price level p_n at t_n depends on instants i_2. After instant t_0, all time up to t_1 including of course t_n is made up of instants i_2. That is, from t_0 to t_1 supply and demand of the commodity are separate factors which are adjustable to each other by variation of the price level.

2. Determination of national income level

Income realized in period $t_0 - t_n$ is represented by the entire area *abed*.

Can we define excess demand for interval $t_0 - t_n$?

—If income had remained constant at level Y_0 from t_0 to t_n, the total amount of output for that period would be represented by area *abcd*. From this seems to result the definition of a positive excess demand equal to the difference between the values of the two areas, *abed* — *abcd* = *dce*.[2]

—If the level of income had increased more than it actually did during period $t_0 - t_n$ (curve III), the excess demand produced during interval $t_0 - t_n$ would have been negative and equal to the difference between the

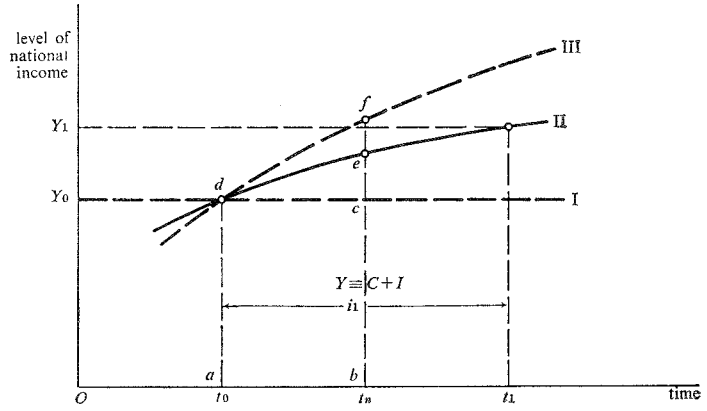


Fig. 9

values of the two areas, *abed* and *abfd*, or *def*.[2] But does not the preceding definition of excess demand imply a contradiction? If realized income were represented by *abcd* instead of *abed*, the corresponding demand would be represented by *abcd* and not by *abed*, since for all realized income identity $Y \equiv C+I$ prevails. Consequently, if realized income were represented by area *abed*, the corresponding demand would necessarily be represented by the same area *abed*. Likewise, it is logically impossible to assume (i) that income extends to *abfd* and (ii) that demand remains at value *abed*.

This conclusion may be expressed differently. Curves I, II, III can all represent the dynamic level of income.[3] How can we choose among them? By analogy with price analysis, the choice would seem to lie with the curve representing equilibrium between supply and demand, where Y always equals $C+I$. But this criterion is not operative, since in all three curves Y equals $C+I$ for each and every point. If we choose curve II, we must do so for reasons other than to satisfy the equation. Whatever the value of the income assumed to be realized in real time, the corresponding demand necessarily equals it, since by definition $Y \equiv C+I$ for any realized income. For every point defined on curve II, the level of excess demand is indeed nil. But excess demand is likewise nil for all other conceivable curves I, III, etc. Excess demand offers no criterion of choice. For excess demand to mean something in the context of

national income, *realized* income would have to deviate from the very demand which defines it, a clearly absurd proposition. Thus, as dynamic excess demand is meaningless in national income analysis, curve II must be explained by a method different from the one used in price analysis.

PERIOD ANALYSIS

19. If we now divide time into equal periods, days, for instance, we have a time unit which suits both price and income theories.

—Purchases and therefore prices occur on the daily market.

—The level of national income cannot be constant from midnight to midnight inside one day. It is likely that fewer people are at work at 10 p.m. than at 10 a.m. So it seems justifiable to select the daily *average*. Suppose, for example, that 50 workmen are employed between midnight and 8 a.m., then 200 from 8 a.m. until 4 p.m., and 50 again from 4 p.m. until midnight. In that case, the income level produced during the day obviously corresponds to the *constant* use of 100 workmen.

According to this method, the level of national income cannot possibly vary within the time unit chosen. If it does vary, it must be from one day to the next.

Period analysis yields the same results as continuous analysis.

20. *Period analysis of price levels.*

1° Basically, the continuous curve $s=d$ could be conceived even in period analysis.

2° Period analysis lends itself better to an economy which has new production daily.

3° If a hypothetical economy produces only one commodity, say oranges, that commodity is simply produced every day.

4° If, on four consecutive days, only one purchase takes place each day, four amounts of the 'single commodity' will be sold successively. Dynamic analysis of price level is then slightly altered.

The only difference is the following. Let s_n and d_n represent respectively the supply and demand of goods produced for the market of J_n. Identity $s_1 \equiv d_1$ appears only once. In general, identity $s_n \equiv d_n$ occurs only once in real time for $n=0, 1, 2, 3, \ldots$

This suggests an interesting result. Every marketable commodity is governed by supply and demand factors which coincide only once, at the exact moment of exchange. At all instants of real time preceding every purchase, the two factors of demand and supply are separate.

We can therefore conclude that time is divided into two phases: (i) *phase ex ante which corresponds to the adjustment between supply and demand*, and (ii) *phase ex post when supply and demand coincide in realized purchases.*

—*Phase ex ante* includes an infinity of instants, which precede exchange. *Ex ante*, we have $s_n \neq d_n$, which is the very definition of excess demand.

—*Phase ex post* involves only the single instant where purchase is realized. *Ex post*, $s_n \equiv d_n$.

Thus, any marketable commodity goes through two time phases: (i) phase *ex ante* where supply and demand of the goods are adjusted to each other, and (ii) phase *ex post* when the two factors instantaneously merge into one and the same transaction.

21. *The level of national income in period analysis.*

1° Time is divided into an unbroken series of equal intervals, into days, for example. In continuous analysis, national income may continually vary. In period analysis, income cannot vary within the chosen time unit, but only by sudden jumps from day to day.

2° A graph will help us solve the following problem. *Can the assumed dynamic variation in national income be ascribed to excess demand?* If so we will have found the explanation of the variation. And if it cannot be attributed to excess demand, although we will not know the causative factor of variation, we will know positively, and this is the real point, that total excess demand cannot explain this variation.

Positive increase of income is shown on the graph by the shaded areas.

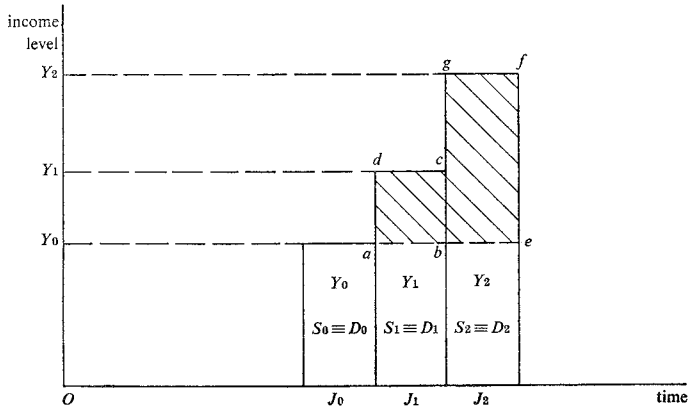


Fig. 10

The time dimension of income derives from a proposition which is almost tautological: one day's production is necessary to produce a day's income. From this, we see that real time includes a series of identities between total supply and total demand: $S_0 \equiv D_0$, $S_1 \equiv D_1$, $S_2 \equiv D_2 \ldots$. In real time, then, no disparity can possibly occur between income and its corresponding demand. Disparities which appear on the graph are but between demand and demand, or between successive incomes. Never is it conceivable that one may observe even the smallest difference between any income and its corresponding demand. Thus, during the whole interval $J_0+J_1+J_2$ and indeed during any length of time whatever, total demand and total supply are identical factors.

$$S_0+S_1+S_2 \equiv D_0+D_1+D_2.$$

Clearly, then, in real time total demand can never be 'ahead of' or 'behind' total supply. The two factors are always and necessarily equal, with the result that excess demand $D \gtrless S$ is a contradiction in terms.

3° If income were to remain constant at level Y_0 during day J_1, could not total demand, nevertheless, in J_1 reach level Y_1? In this case, $D > Y$; for demand would equal Y_1, whereas income would only equal Y_0. But this involves contradiction. Demand cannot vary from J_0 to J_1, if we assume that income remains constant during that interval. Any realized income being, by definition, identical with the corresponding total demand, we cannot logically admit constant income if demand is increasing. Area *abcd* does not represent the excess of demand in J_1 over supply in J_1, but the excess of income in J_1 over that in J_0. Similarly, area *befg* shows no excess demand. It denotes the dynamic increase of income over income, or of demand over demand. At no moment can the slightest difference be observed between income and the corresponding demand. No total excess demand can ever appear, so that total excess demand can never explain income variation. In the theory of income, the concept of excess demand is unjustifiably borrowed from the theory of prices.

4° Analogy between the two theories of price and income could hold only if the distinction between the two time phases were to apply to the determination of national income. But no instants on the time axis do coincide with phase *ex ante*. There is not a single instant to be found on the time axis where $S \equiv D$ cannot be observed. Identity $S \equiv D$ can never be disproved, for it results from the very definition of national income. In the theory of income, phase *ex post* extends over

the whole period, whether in the short run or in the long run. In price theory phase *ex post* only covers a finite number of instants. The difference comes from the fact that price (purchase) is an instantaneous transaction, while income can only be produced in a finite time interval.

*
* *

The income of an N-day period is realized in the total period of N days. But N days are not necessary to realize all the purchases occurring during the N-day period.

The conclusion is already known. During the whole N-day period, where N can equal 100 years, no moment exists during which Income and Demand are not identical. Constantly identical, they passively follow in time the variation of national income level. If the level of income actually varies in the course of a century, for example, as it no doubt does, this variation can never be due to the adjustment of total income to total demand. Demand cannot call forth Supply, for these two factors are merely two aspects of one and the same thing. National income determination is not amenable to dynamic analysis.

B. CRITICAL APPRAISAL OF THE DOMINANT SCHOOL

"... I think that most economists feel that short-run macroeconomic theory is pretty well in hand. ... The basic outlines of the dominant theory have not changed in years. All that is left is the trivial job of filling in the empty boxes, and that will not take more than 50 years of concentrated effort at a maximum."

R. M. SOLOW[4]

Economists of the future, up to the year 2000 and beyond, will only have to perfect in detail a theory whose fundamentals will already be proved and certain. This insignificant task will be incumbent on minds born too late, after the edifice of science is already constructed. The only work left will be, so to speak, to furnish the house.

If this prospect is gloomy, the reality is, happily, neither so circumscribed nor so irksome. Works too must perish. New generations will find the coast clear. Old houses fall into ruins and must be reconstructed from top to bottom.

But enough of this allegory suggested to us by the great economist SOLOW. We shall now attempt to show that the so-called Keynesian

income theory depends on the deceptive analogy which we have just criticized in the first part of this chapter. And since the analogy obviously fails, it is evident that the dominant theory misinterprets the very basis of macroeconomics.

Of the four authors to be discussed, Paul A. SAMUELSON, Abba P. LERNER, R. G. D. ALLEN and Dennis H. ROBERTSON, the first three are generally considered to be orthodox Keynesians. SAMUELSON and LERNER are founders; ALLEN represents the dominant theory in its most recent state (1967) with a very high level of didactic accuracy. As for ROBERTSON, we know that he rivalled KEYNES and is therefore of special interest in this study. Can the basic Keynesian identity $Y=C+I$ really be challenged?

PAUL A. SAMUELSON

"II. THE HEART OF INCOME ANALYSIS

"By definition, *national income* (at market prices), Y, can initially be set equal to the sum of consumption expenditure, C, and *net investment*, I:

$$Y=C+I$$

"If KEYNES had stopped with this identity, we should be left with an indeterminate system. In his simplest model of income determination, he added the following two hypotheses: (*a*) consumption is a function of income, and (*b*) investment may provisionally be taken, at any one time, as a constant. Mathematically, these relations may be written

$$C=C(Y) \text{ and } I=\bar{I}$$

"When we substitute these into our first identity, we come up with the simplest Keynesian income system:

$$Y=C(Y)+\bar{I} \tag{1}$$

This is a determinate system, being one equation to determine one unknown variable. While much of the anti-Keynesian and Keynesian world was still arguing over the tautological character of the Keynesian concepts, Professor HANSEN had quickly cut through the non-essentials to isolate the critically important role of the propensity-to-consume schedule, as embodied in this fundamental equation.

"Equation (1) is crucially important for the history of economic thought. It is the nucleus of the Keynesian reasoning. If it *in no way* gives insight into the analysis of employment, then the Keynesian system

is sterile and misleading. In its oversimplification, this relation must be compared with two other seminal single equations which contain by implication much of the remainder of economic theory: namely the equating of supply and demand to determine market price,

$$D(p) - S(p) = O;$$

and the determination of a firm's best output, q, (or anything else) by the condition that its profits, π, be at a maximum through the balancing of the effect of any decision on *total revenue*, R, and *total cost*, C,

$$\frac{d\pi}{dq} = \frac{dR(q)}{dq} - \frac{dC(q)}{dq} = O"^{5}$$

SAMUELSON postulates two openly contradictory propositions.

1° National income is equal to $C+I$. This equality results from a definition: "If KEYNES had stopped with this identity,..." So here we have a definitional identity.

2° Equation $Y=C+I$ cannot obtain for all values of Y but only for the equilibrium value of national income. In other words, the two terms of equation $Y=C+I$ are independently determined, so that equality between them denotes a particular value of Y, $Y=Y_0$, which defines income equilibrium. For any value of Y different from Y_0, equation $Y=C+I$ does not hold.

The two propositions 1° and 2° are contradictory for the simple reason that an equality is either conditional or necessary. Either equality is conditional and therefore not necessary, since it is deficient when the required condition is not satisfied, or else it results from a definition and is necessary since the two terms of a definition must logically be equal.

We cannot posit that $Y=C+I$ is simultaneously a definition and a condition of equilibrium.

But SAMUELSON obviously knows the difference between a definitional and a conditional equality.

The profits of a firm, π, grow when production makes receipts, R, increase more than costs, C. Thus the best possible output can be determined at the exact point where the increase in receipts equals that of costs. Equality

$$\frac{dR(q)}{dq} = \frac{dC(q)}{dq}$$

is conditional and not necessary.

Is equality of Y and $C+I$ likewise conditional? As a matter of fact, it is not. It is a definitional identity. National income is equal to the value of current output which can only be measured by the sum of consumption and net investment purchases. This is why SAMUELSON defines national income Y as being equal to $C+I$. "By definition, *national income* (at market prices), Y, can initially be set equal to the sum of consumption expenditure, C, and *net investment*, I: $Y=C+I$. If KEYNES had stopped with this identity, we should be left with an indeterminate system."

Now, since $Y=C+I$ is an identity, the link between $C+I$ and Y is as strong as the link between Y and Y or between $C+I$ and $C+I$.

$$Y \equiv C+I$$

$$Y \equiv Y$$

$$C+I \equiv C+I$$

The last two identities are mere tautologies. The first identity, on the contrary, is a proposition, furnishing positive information. Although Y and $C+I$ designate the same thing, it has two sides, supply and demand. All three equations are nevertheless identities in the proper meaning of the word. *The two terms of each equation indicate the same reality.* If Y were to stand for one concept and $C+I$ for another, the two terms could not be identical.

SAMUELSON proposes to determine Y by $C+I$, $C+I$ being known as a function of Y. But how is this tenable? SAMUELSON's 'solution' really amounts to finding the unique value of Y for which $Y=Y$. How could the balance between the two terms of an identity be a determining criterion?

Paul A. SAMUELSON could not possibly have ignored the logic of this basic distinction between a condition of equilibrium and an identity. Indeed, his own words prove his awareness of this distinction. "Being such an identity and not a condition of equilibrium, it cannot possibly help to determine the level of the rate of interest or level of income."[6]

How can SAMUELSON take $Y=C+I$ for an identity and simultaneously for a condition of equilibrium? His own basic equation, which he calls "seminal," furnishes the answer.

$$D(p)-S(p)=O$$

Since the equation of supply and demand of a commodity is a condition of equilibrium as well as an identity, why would not equation $Y=C+I$ reveal the same double nature?

No equation *in any field* can be defined simultaneously as an identity and a condition of equilibrium. It must be one or the other. If it has both meanings, they must be successive. They cannot conceivably be simultaneous.

Thus d and s are at first distinct quantities and must be equalized before transaction takes place. Later, at the instant of exchange, d and s become the two terms of an identity. During the *ex ante* phase, demand and supply progressively adjust. In this way, price levels vary so as to permit exchange. In any purchase, supply and demand are opposite sides of a single transaction. SAMUELSON agrees: "It is, of course, true that there are two sides to every transaction, and that... the value of sale must be equal to the value of purchase. But it is not true,... that over an interval of time the value of an existing asset cannot change or cannot be changing. It is only by concentrating upon the 'instant' of sale that the cancelling out emerges."[6]

As regards price theory, we must differentiate between the two families of instants i_1 and i_2. Logically we can write

$$i_1 : s \equiv d$$

$$i_2 : s \not\equiv d$$

since

$$i_1 \not\equiv i_2$$

The two relationships

$s \not\equiv d$: condition of equilibrium

$s \equiv d$: identity

never coincide, for they hold at different times. The argument would be illogical only if we stated the two simultaneously.

The theory of income does not differentiate two families of instants, for income, unlike prices, cannot be determined instantaneously. If the level of national income be known and represented by a continuous curve in function of time, each point on this curve corresponds to an instant in real time. Now each point furnishes two pieces of information: (i) its distance above the horizontal axis indicates the instantaneous national income *level*; (ii) the slope of the tangent at this point gives the instantaneous rate of increase of income level. The two quantities, income level and rate of increase, positive or negative, can be defined instantaneously. But a third quantity, *national income itself*, as opposed to the *level* of national income, is nil at each point. At any point, whatever the instantaneous level of national income, the measure of income is

zero. For income must have a time dimension. We can speak of yearly, daily, hourly income, or of a second's income, if we like. But we cannot speak of an instant's income. The theory of income does not differentiate the two families of instants i_1 and i_2. Consequently, it is a flat contradiction to write

$$Y \neq C+I\text{: condition of equilibrium}$$

$$Y \equiv C+I\text{: identity}$$

The two necessarily conflict, since both occur at the same instants i_1. The mistake is due to a misleading analogy, which holds good only when real time can be divided into two series of instants. When the distinction is not firm (and it is not in the case of national income), the analogy breaks down.

SAMUELSON and numerous other authors use the 45° diagram.

"Graphically, the simplest Keynesian equilibrium can be shown on a by now familiar 45° line diagram. On the vertical axis the consumption function, $C(Y)$, is plotted against income. Investment is then superimposed onto consumption. The two together constitute the right-hand side of equation (1). The left-hand side, Y, is simply income itself plotted against income, or in short a 45° line. The intersection of $C(Y)+\bar{I}$ with the 45° line gives us our simplest 'Keynesian-cross,' which logically is exactly like a 'Marshallian-cross' of supply and demand."[7]

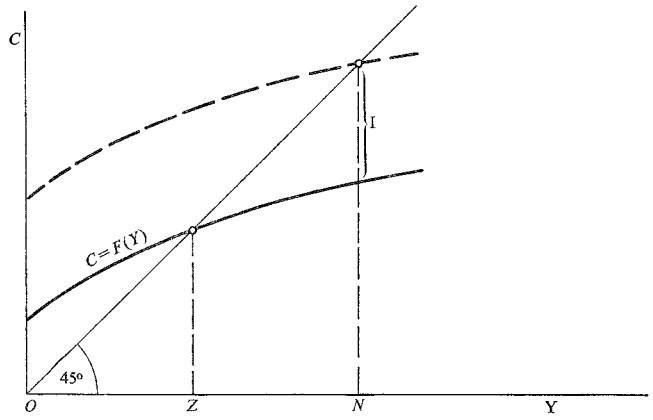


Fig. 11

(SAMUELSON, *op. cit.*, p. 1115)

We can see that equality between Y and $C+I$ occurs only for national income value equal to ON. For any other value of Y, the corresponding demand is greater or smaller than income.

The graph brings out the weakness of the argument. For zero income, the sum of consumption and investment cannot be positive, for that sum defines national income. If the sum is positive, so is income, necessarily. Generally speaking, value Y always results from the addition of values C and I, so that the sum of the two variables, consumption and investment as a function of income, is represented by the 45° line.

Since national income is a known quantity before consumption and investment are determined, functions $C=C(Y)$ and $I=I(Y)$ are not independent.

—Suppose $I=\bar{I}$. The consumption function is implicit, for necessarily $C=C(Y)=Y-\bar{I}$.

—Suppose now that we find a consumption function of any form, linear or not. Investment cannot possibly be constant, for if it were, we would have $I=\bar{I}$ only for the very special function $C=Y-\bar{I}$. For any other form of the consumption function, we have $I=I(Y)=Y-C(Y)$, where I is not a constant.

The sum of consumption and investment always equals income. Total demand as a function of income is represented by the 45° line. The total supply and total demand curves thus coincide and their points of 'intersection' are infinite in number. Clearly then, national income cannot be determined in this way, since any conceivable income is, according to this criterion, an equilibrium income.

Income and Demand being identical by definition, their equality cannot be a criterion of equilibrium.

If national income increases in time, which is obviously not ruled out, its growth cannot in any way be due to adjustment between total supply and total demand. No adjustment between these quantities can occur in real time, so that the law governing dynamic income variation remains completely unknown. SAMUELSON's equation (1) "crucially important for the history of economic thought",

$$Y=C(Y)+\bar{I} \tag{1}$$

means $Y=Y$ and nothing more. Given $I=\bar{I}$, the consumption function is known: $C=C(Y)=Y-\bar{I}$. We obtain

$$Y=C(Y)+\bar{I}=Y-\bar{I}+\bar{I}=Y. \tag{1}$$

National income is the unknown Y. Total demand $C+I$ is the *same unknown* seen from the opposite side. Thus in SAMUELSON's system

$$Y=C+I$$
$$C=C(Y)$$
$$I=\bar{I}$$

income remains indeterminate.

There are only two unknowns, since $C+I$ is the same unknown as Y. So, if two of the three quantities Y, C and I are known, the third must also be known. Unknowns number two, and not three as SAMUELSON takes for granted.

How many independent equations are there? Equation $Y=C+I$ is one and the same unknown as seen from two standpoints. Identities provide no determining factors. On the other hand, equation $C=C(Y)$ is additional, for it states C as a function of Y. Finally, equation $I=\bar{I}$ or, more generally, $I=I(Y)$, is implicit in equation $C=C(Y)$, since C and I are complementary in Y. Therefore there remains one and only one determining equation.

Since the single equation has two unknowns, it cannot be solved. For example, suppose function $C=1/2\ Y$. Then we must have function $I=1/2\ Y$. Applying these two functions to the definition of national income, SAMUELSON's result is obtained:

$$Y=1/2\ Y+1/2\ Y=Y$$

which leaves income utterly undetermined.

Professor Paul A. Samuelson's explanation leaves income totally indeterminate. If a consumption function can be ascertained, we then know how to divide any given income between consumption and investment spending. But income thus divided remains none the less completely undetermined.

In his book *Economics and Employment*,[8] LERNER also proceeds by counting equations and unknowns.

ABBA P. LERNER

To clarify the question, LERNER gives two examples.

First case. No net investment occurs.

Second case. Expenditures for net investment are added to current consumption.

We can study these two cases in the author's text. In both instances, we find the following basic contradiction. Although the author writes

identity $Y \equiv D$, he allows demand D to deviate from income Y, even though he has himself severely censured this very contradiction.

1. *First case:* $Y=C$

"To simplify our example to the extreme, let us suppose that no investment takes place at all so that we can for the time being leave it out of the picture. Income is then created only by the spending on consumption goods and is equal to consumption. This gives us our first equation, $Y=C$, where Y stands for income and C stands for consumption. (The letter Y is used to represent income because the initial I has traditionally been used for investment and some other letter had to be substituted.)

"The second equation is provided by the *propensity to consume*, which is the way in which consumers adjust their consumption to their income. Let us suppose that $C=40+{}^2/_3\,(Y-40)$. This means that the propensity to consume is such that the people in the country will consume 40 billion dollars a year plus two-thirds of any excess of their income over 40 billion dollars.

"We have two equations:

(1) $$Y=C$$

(2) $$C=40+{}^2/_3(Y-40)$$

"The first equation enables us to substitute Y for C in (2) obtaining an equation with a single unknown:

$$Y=40+{}^2/_3(Y-40)$$

"Solving this by easy steps we have

$$Y=40+{}^2/_3(Y)-{}^2/_3(40)$$

$$Y-{}^2/_3(Y)=40-{}^2/_3(40)$$

$$\frac{Y}{3}=\frac{40}{3}$$

$$Y=40"^{9}$$

First, we shall simply accept LERNER's consumption function $C=40+{}^2/_3(Y-40)$, that is equation (2), leaving out the definition of national income, equation (1). How far will the consumption function take us? Once we know the exact inference to be drawn from equation (2), and only then, we can introduce equation (1), in the case where investment

is zero, as LERNER assumes. How does the introduction of identity $Y=C$ complete or modify the consumption function?

Consumption function.

Consumption can be calculated for any given incomes. This function also states the value of Y for which $Y=C$. When consumption is equal to $40+{}^2/_3(Y-40)=\$40$ billion, variables Y and C are equal, according to the chosen consumption function. For any other value of C, C is different from Y. Or, identically, for any other value of Y, Y is different from C.

The reader may draw the graph of line $C=40+{}^2/_3(Y-40)$, with income measured on the horizontal axis and consumption on the vertical axis. The point of equality between Y and C lies obviously on the 45° line. For any value of Y above or below 40, the corresponding consumption is different from Y.

Point of intersection P between the 45° line and the line representing the consumption function satisfies both equations, $C=C(Y)$ and $C=Y$. Here, this second equation, $C=Y$, is not however the definition of national income, *but is simply the equilibrium condition between the two variables as defined separately, income and consumption.* We should note also that point of intersection P indicates only a certain value of C and Y, when consumption and income are equal to \$40 billion. We do not claim that consumption must necessarily reach this sum, but that if it does income will be equal to it.

We now bring in the definition of national income. Is the consumption function thereby modified?

The moment the definition of national income, $Y\equiv C$ (I being zero), is introduced, LERNER's equations (1) and (2) clash. The two equations

$$Y\equiv C \tag{1}$$

$$C=40+{}^2/_3(Y-40) \tag{2}$$

are mutually inconsistent.

Indeed, if Y is *necessarily* equal to C, C is necessarily equal to Y. And if consumption is *necessarily* equal to income, the consumption function cannot be written $C=40+{}^2/_3(Y-40)$, but must be written

$$C=Y.$$

Once the definition of national income is introduced, the consumption function must comply with this definition. In an economy where net

investment is zero, all income is necessarily consumed, since consumption is then the very definition of national income. Is LERNER's analysis not rather unusual? He starts with definition $Y \equiv C$. "Income must be exactly equal to spending simply because the two words refer to the same thing looked at from different points of view." (p. 64) In other words, Y and C refer to the same object. But after stating identity $Y \equiv C$, LERNER goes on to define consumption as a function of an income which it does not necessarily equal. For an infinite number of values of Y, equation $C = 40 + {}^2/_3(Y - 40)$ permits a consumption unequal to the corresponding income. In LERNER's analysis *the identity of Y and C thus goes along with the non-identity of C and Y.*

In reality, identity of income and consumption curbs the buyers' propensity to consume. LERNER fails to draw the logical consequence.

"The public may, for instance, have been led to believe that they were earning 46 billion dollars and therefore, in accordance with their propensity to consume, spent 44 billion dollars on consumption $[40 = {}^2/_3(6)]$. If they do that their income will in fact be not 46 billion dollars but 44 billion dollars, and they will be consuming more than is indicated by their propensity to consume. They will be consuming 44 billion dollars instead of the $42\,{}^2/_3$ billion dollars $[40 + {}^2/_3(44 - 40)]$ that they should spend out of an income of $44 according to their propensity to consume."

LERNER here followed a rule which no doubt is both clear and correct. If the propensity to consume is such that C is different from Y, the propensity is necessarily modified, so that consumption again equals income. Identity $Y \equiv C$ overrules the consumption function. Assuming investment to be zero, the consumption function is then simply $C = C(Y) = Y$, for it must comply with identity $Y = C$.

LERNER gives two examples of the fact that the consumption function must respect the definition of income. If income is $44 billion consumers spend $44 billion, whereas the propensity to consume would have given only $\$42\,{}^2/_3$ billion. If income were $38 billion, consumption would be equal to $C = 40 + {}^2/_3(Y - 40) = \$38\,{}^2/_3$ billion according to the schedule of the propensity to consume. In fact, the function is overruled, for it must comply with identity $Y = C$. So consumption is reduced to $38 billion. "A belief that income was being earned at the rate of 37 billion dollars would cause the population to consume at the rate of 38 billion dollars. Their income would then be 38 billion dollars and not 37 billion and they would not be consuming the $38\,{}^2/_3$ billion dollars which the propensity to consume tells us they should consume if their income were 38 billion dollars $[40 - {}^2/_3(40 - 38)]$". (p. 70)

In short, when function $C=C(Y)$ diverges from $Y=C$, that is, from $C=Y$, it is forcibly brought back to this equality. The function cannot be written $C=40+{}^2/_3(Y-40)$ as LERNER has it, for then Y would differ from C for any value of $Y \neq 40$. Thus, to avoid all divergence, since income must be identified with expenditures, the function must be written $C=C(Y)=Y$.

The difficulty in LERNER's argument lies in the ambivalent use he makes of the consumption function. In each of his examples he first endorses the law of propensity to consume and then immediately invalidates it.

LERNER's first example includes two phases.

1° *Statement of the consumption function.* Consumers expect an income of \$46 billion dollars. According to the propensity to consume, C must then equal \$44 billion. If consumers actually spend \$44 billion, the law expressed by the propensity to consume is thus confirmed.

2° *Invalidation of the consumption function.* Since consumers spend \$44 billion, national income amounts to \$44 billion. Thus C is equal to Y and not to $40+{}^2/_3(Y-40)$. As a result, the law of propensity to consume is invalidated.

LERNER cannot have it both ways. Is the law $C=40+{}^2/_3(Y-40)$ valid or invalid? The answer is clear. The law is applied to a purely hypothetical income and therefore can neither be supported nor refuted. The law of propensity to consume, or the consumption function, can only be experimental. But no experiment could show consumption as a function of a non-existent, purely hypothetical income. If facts show $C=40+{}^2/_3(Y-40)$ for any realized income, surely the same law can then be applied to hypothetical incomes, for instance to the incomes of \$46 billion and \$37 billion in LERNER's examples. But if the law were to be invalidated by experimentation, so that it could not be upheld for realized incomes, how could it hold for purely hypothetical incomes? In other words, the logical position is (2°) and not (1°).

As soon as the sum actually spent by consumers is known, the corresponding income is given by equation

$$Y=C$$

and not by $Y=\frac{3C-40}{2}$ as LERNER states.

For C *realized* = \$44 billion, income *realized* is \$44 billion and not $(3 \cdot 44-40) : 2=\$46$ billion. And for C *realized* = \$38 billion, income *realized* is \$38 billion and not $(3 \cdot 38-40) : 2=\$37$ billion. The fact that LERNER approves these results (\$44 instead of \$46 and \$38 instead of 37 billions of dollars) shows that he is dissatisfied with his own law,

$C=40+{}^2/_3(Y-40)$, and replaces it by $C=Y$. Willy-nilly, he completely destroys in his own examples the consumption function he posits.

The graph on page 78 of *Economics of Employment* may serve as a conclusion to the first case ($Y=C$).

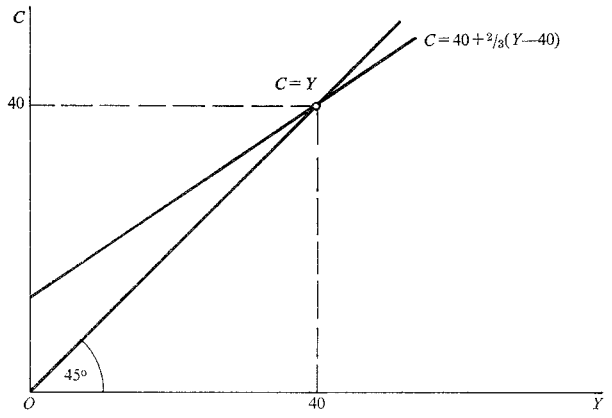


Fig. 12

In short, the graph represents *two* consumption functions.

Function $C=40+{}^2/_3(Y-40)$ would be possible if the definition of income were not $Y=C$.

But since the definition of income is indeed $Y=C$, assuming investment to be zero, the consumption function is necessarily represented by line $C=Y$ (the 45° line).

LERNER's reasoning reflects the dominant theory according to which national income and total expenditures are identical and yet equal only in a state of equilibrium.

"The equality between income earned and money spent is one of the most fundamental propositions in the theory of the determination of income and employment. A failure to recognize the equality or a supposition of the possibility of even a momentary inequality between them involves error and contradiction and completely destroys any arguments that permit it.

"This may seem rather strange because the equality between income and spending is one of those unconditional and inevitable equalities that are sometimes called identities or tautologies. It does not really make any statement about the actual world in the sense that it tells something about it which we might have supposed to be otherwise. But although the equality or identity of spending and income does not give us any infor-

mation about the actual world, it is extremely useful in checking on arguments which do purport to tell us something about the actual world. If any argument should imply that income was greater or smaller than spending, even if only for the smallest period or only by the smallest amount, then that argument is no good." (p. 65)

The identity of national income and total spending is indeed a basic proposition in macroeconomics. However, LERNER does not consistently stick to his definition. Identity $Y=D$ is undebatable. Hence, total demand cannot under any circumstances be different from the corresponding income. Demand as a function of income must be written $D=Y$. But, instead of concurring with this result, LERNER thinks that D can be unequal to Y, while Y must be identical to D. Indeed, where total expenditure is $C=40+{}^2/_3(Y-40)$, D is different from Y for a whole series of national income values. Now we can judge LERNER by his own words. "If any argument should imply that income was greater or smaller than spending, even if only for the smallest period or only by the smallest amount, then that argument is no good." (p. 65) The argument "is no good" which admits any possible inequality between total expenditures and national income. The main criticism of the dominant theory is obvious. Identity of Y and D being indisputable, total demand as a function of income must be expressed by $D=Y$. *In particular, in an economy where net investment is zero, buyers necessarily purchase for consumption all goods produced. Thus purchases can neither exceed output nor can output exceed purchases.* "This may seem rather strange because the equality between income and spending is one of those unconditional and inevitable equalities that are sometimes called identities or tautologies." (p. 65) Equality between production Y and spending D, or C when investment is zero, is "inevitable and *unconditional*" in LERNER's own terms. From this we can draw the tautological conclusion: an unconditional equality like $Y\equiv C$, I being zero, is true at any moment. The dominant theory, then, combines the two inconsistent statements:

(1) $Y\equiv D$: identity

(2) $D=Y$: condition of equilibrium.

Since equation (1) is "unconditional," so is equation $D=Y$. Therefore spending is equal to income for any value of Y and not only for a single value determined at the "equilibrium point" between income and total spending. Thus the idea of the dynamic equilibrium of national income must be discarded.

2. *Second case:* $Y=C+I$

"Algebraically we now have three equations:
(1) $Y=C+I$ (I stands for investment. The consumption and the investment between them create all the income.)
(2) $C=40+{}^2/_3(Y-40)$ (The propensity to consume.)
(3) $I=10$ (The assumed level of investment.)

Again we can proceed to solve the simultaneous equations by simple steps. Combining (3) and (1) we get

$$Y=C+10$$

Substituting this value of Y in (2) we get

$$\begin{aligned} C &= 40+{}^2/_3(C+10-40) \\ &= 40+{}^2/_3(C-30) \\ &= 40+{}^2/_3(C)-{}^2/_3(30) \\ C-{}^2/_3C &= 40-{}^2/_3(30) \\ \frac{C}{3} &= 40-20 \\ C &= 60 \end{aligned}$$

and Y, which is equal to $C+I$, is 70." (p. 72)

We shall not develop the criticism at length, since the conclusions reached in the first case can all be applied again.

(i) If the definition of national income were not $Y=C+I$, the consumption function could assume any form whatsoever. We could write, if we liked, $C=40+{}^2/_3(Y-40)$, even though investments should constantly amount to $10 billion per year.

(ii) Since $I=\bar{I}=10$, the propensity to consume cannot be $C=40+{}^2/_3(Y-40)$, but must necessarily be $C=Y-\bar{I}=Y-10$.

(iii) If the consumption function is not $C=Y-10$, which obviously depends on consumers' behavior, then we cannot assume that investment is equal to $10 billion per year. We surely have the right to assert that $C=40+{}^2/_3(Y-40)$. However, the immediate result gives investment $I={}^1/_3(Y-40)$.

(iv) Function $D=D(Y)$ necessarily reduces to $D=Y$. On the graphs,[10] the curve of total demand as a function of national income has to coincide with the 45° line.

The author attempts to determine national income by the equalization of total demand and the corresponding income. But this criterion, $D=Y$, does not differentiate among all possible values of Y. By his

own definition, whatever the value of national income, it is always equal to total demand.

Like SAMUELSON, LERNER attempts the impossible, that is, to find the point of intersection of two coincident curves.

R. G. D. ALLEN

Professor ALLEN's book, *Macro-Economic Theory, a Mathematical Treatment*,[11] is based on integral equations (continuous analysis) or on difference equations (period analysis), by means of which he studies the time variation of total quantities like Income, Savings, Consumption, Investment.

His whole system is based upon the hypothesis that national income depends on economic factors occurring in real time which force its level up or down until equilibrium is reached. The basic hypothesis could also be expressed by saying that any disequilibrium affecting national income would trigger an adjustment process up to a new equilibrium position. Thus, the level of national income varies in time until the initial equilibrium is restored.

ALLEN's view is common to the whole so-called Keynesian dominant theory.

How does ALLEN explain a possible disequilibrium between total income Y and the corresponding demand Z? His proof is in two parts. (1) The difference between demand and income, $Z \neq Y$, results from a lag between the production and spending of national income, without which quantities Z and Y would coincide. (2) But ALLEN makes two conflicting assumptions. If no lag is assumed, no difference can arise between total expenditure and income, so $Z \equiv Y$. Yet, Z and Y can have different values, $Z \neq Y$.

1. "The Circular Flow of Income

"Aggregate demand, which we write Z, is made up of consumers' purchases, investment purchases, government expenditure, and perhaps other constituents, e.g. external demand. We must be clear at the outset on the relation between demand Z and income Y on the one hand and output Q on the other. The relation can be viewed as a conceptual one, the circular flow of income in the economy; or it can be put in the framework of the aggregates of national income accounting.

"The circular flow of income is illustrated in Fig. 2.1A in purely schematic terms. It does not matter where we start but we must follow

the direction of the flow of income. If we start with income Y as received, then demand Z follows from plans to spend income, by consumers, business, and government. Demand Z gives rise to output Q, the goods and services produced by business to meet the demand. Finally, from the proceeds of output, the factors of production receive their reward and income Y follows; we are back at our starting-point. All this is in money terms, but usually we fix prices and the flow is in real terms, e.g. in terms of the consumption good, or in aggregates at fixed (base) prices." (pp. 16–17)

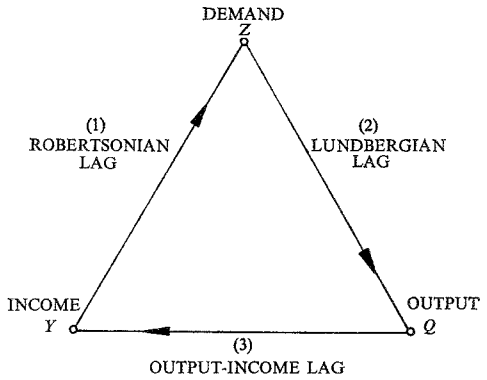


Fig. 13

Like SAMUELSON and LERNER, ALLEN clearly states that total demand Z and national income Y denote the same concept, seen from two opposite standpoints. Nevertheless he contrives to represent them as *two* vertices of a triangle by introducing one or more lags. ALLEN's distinction between Z and Y is governed by the distinction between total demand corresponding to period t and total demand which corresponds to another period, $t+n$. In short, any difference between Y and Z is due to leads and lags. To illuminate this, we can use SAMUELSON's original expression, "saving-investment." "For the system as a whole, saving and investment, as observables, are defined as the same thing... In the above senses, saving and investment may be called for clarity saving-investment."[12] Likewise, Supply-Demand denotes a single reality. Apparently, then, the vertices of ALLEN's triangle stand for one single variable, 'Supply-Demand.' Nevertheless, if the triangle cannot be reduced to a single point, this is because time elapses between succeeding vertices. All three vertices represent the same variable, Supply-Demand, but at different time points.

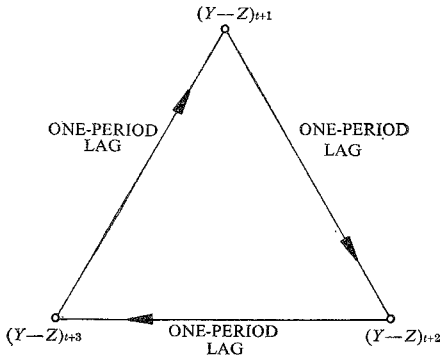


Fig. 14

On the graph, there are only one-period lags. "The simplest form of lag, and the one most often taken, is the delay of a single period." (p. 17) However, ALLEN poses a zero lag between output and factor payments. "Taking a zero lag of the third type, we can write output Q and income Y interchangeably." (p. 17)

Now we can conclude the first part of our criticism. We have shown that Professor ALLEN differentiates between the vertices of his triangle only by introducing lags. To test this, we have only to omit one lag. Thus, a zero lag of the third type, between output and factor payments, results in the fact that Q and Y can be written interchangeably. So we may conclude that when analysis drops all three lags, we are left with a single point of identity between Y and Z.

In an analysis where all lags are zero, no distinction can be made between Income and Demand. These two variables are then merely the opposite sides of one and the same thing, 'Supply-Demand.'

2. Having presented his triangle, ALLEN studies, several pages further on, KEYNES' multiplier, when lags are zero.

Since there is no lag of any type, Income and Demand are identical. As a result, analysis cannot logically distinguish between Y and Z. If these two quantities are shown on the axes, they all lie on the 45° line.

Therefore, Allens (Fig. 2.6) [12 bis] is incorrect, since it permits Z and Y to diverge. Now these variables are equal to each other, for the analysis postulates zero lags and therefore logically implies identity of Demand and Income.

Suppose the two functions to be

$$C = C_0 + bY$$

$$I = I_0 + dY.$$

If coefficients b and d are constants, the functions are linear.

However, the total demand function is of special interest here. This function is represented on Fig. 2.6 by curve OC. It should have been represented by the 45° line drawn from origin O. This is because:

1° The sum C_0+I_0 is necessarily zero. It is true that $Y=C_0+bY+I_0+dY$. But for $Y=0$, C_0+bY+I_0+dY must be equal to 0. Therefore $C_0+I_0=0$.

2° Since the sum C_0+I_0 is equal to zero, only two possibilities remain:

—the consumption function and the investment function include two constants: $C_0=I_0=0$,

—the two constants in the functions add up to zero: $C_0=-I_0$.

3° The sum of coefficients b and d is equal to one, for equation $Y=C_0+I_0+bY+dY$ must hold for any value of Y. Hence we must have $b+d=1$ (when $Y\neq 0$). As a special case, when d is equal to zero, b is equal to one.

Consequently, function $Z=Z(Y)$ must be written

$$Z=\underbrace{C_0+I_0}_{0}+\underbrace{(b+d)}_{1}Y.$$

$$Z=Y.$$

Point P on the diagram is meaningless. Value OM is no more an equilibrium value of national income than any other value of the variable. In other words, equality between total supply and total demand is no criterion of equilibrium. Analyzed by this method, national income remains entirely indeterminate.

Corollary. Curve OC' is just as objectionable as curve OC.

"The effect of an increase in the propensity to save (i.e. a decrease in the propensity to consume) is also traced most easily from Fig. 2.6. The curve C becomes everywhere less steep and shifts downwards to the position C'. The equilibrium position becomes P' and the equilibrium income level falls from OM to OM'. However, since $P'P$ is the 45° line, the fall in Y is matched by an equal fall in C and $S=Y-C$ is unchanged. This must be so, since $S=A$, the given autonomous expenditure. Hence, if there is a greater propensity to save, the aggregate of saving remains fixed and it is out of a smaller level of income." (p. 28)

ALLEN's example $I=I_0$ implies that there is no induced investment. Induced incomes are equal to bY. But d being zero, b is equal to 1.

Therefore the propensity to consume cannot be made to vary; it is constant. In ALLEN's hypothesis, where induced investments are zero, all additional income is necessarily consumed. If increases in income, ΔY, were not always followed by equal consumption increases, identity $Y=Z$ would be invalidated, which is absurd. In the general case where induced investments are positive, we find $\Delta C+\Delta I=\Delta Y$. The sum of the coefficients, $b+d$, remains equal to one. This is necessarily so, since all income is consumed or invested. This is the immediate outcome of identity Z (or $C+I) \equiv Y$.

Is ALLEN's mistake to be attributed to the mathematician or to the economist?

In mathematics, equation $Y=C+I$ can be a condition of equilibrium, but not in economic analysis. Professor ALLEN holds to *the identity of Income and Expenditures when delays in adjustment (lags) are zero.* "Taking a zero lag of the third type, we can write output Q and income Y interchangeably." Substituting the first lag for the third, we find: "Taking a zero lag of the first type, we can write income Y and demand Z interchangeably." Consequently, symbols Y and Z are *interchangeable*, that is, they represent different sides of the same variable. Equation $Y=C+I$ or $Y=Z$ is not a condition of equilibrium but an identity, a fact which ALLEN recognizes perfectly.

From here on, the mathematician and the economist agree. Consumption and investment expenditures are complementary factors in national income. The sum of expenditures cannot be greater or less than national income. Hence, function $Z=Z(Y)$ is represented by the 45° line. National income is thus totally indeterminate. Whatever the form of the consumption and investment functions, they always add up to $Y=C+I$.

But an interesting question remains unanswered. Identity of Income and Demand depends, according to ALLEN's theory, on the absence of lags between production and total demand. Can equation

$$Y=C+I$$

be considered a condition of equilibrium when lags are positive?

Certainly not. The great economist Dennis H. ROBERTSON was misled by this very problem.

DENNIS H. ROBERTSON

We shall examine: (1) the statement of ROBERTSON's thesis; (2) the first criticisms; (3) the exact meaning and scope of the 'Robertsonian intervals.'

1. "I assume the existence of a period of time, to be called a 'day,' which is finite but nevertheless so short that the income which a man receives on a given day cannot be allocated during its course to any particular use. A man's disposable income—the income about which the question arises on any particular day as to whether it shall be 'saved' or 'spent'—is thus the income received not on that day but on the previous one. A man is said to be *saving* if he spends on consumption less than his disposable income."[13]

If time is divided into small "significant intervals,"[14] equation $Y=C+I$ is no longer an identity, for equality of Income and Expenditures is no longer necessary.

The income produced during period t is Y_t. Resulting from consumption and investment expenditures, income Y_t is identical with $(C+I)_t$, the sum of expenditures for period t. This income is not available in period t but only in period $t+1$, where the unit represents ROBERTSON's "finite and nevertheless rather short space of time." Expenditures from income Y_t are the consumption and investment of period $t+1$ if, like consumption, investment is lagged on income.

Obviously the two identities

$$(C+I)_t \equiv Y_t$$

$$(C+I)_{t+1} \equiv Y_{t+1}$$

do not produce identity $Y_t \equiv (C+I)_{t+1}$.

On the contrary, positive or negative differences may arise between successive incomes. Thus it is possible to find

$$Y_{t+1}=(C+I)_{t+1} \neq Y_t=(C+I)_t.$$

Why are incomes of the two periods $t+1$ and t different? ROBERTSON attributes the difference to the *behavior of buyers.* Income produced inside period t becomes available in period $t+1$. Consequently, the behavior of income holders is such that they spend either (i) all their available income, or (ii) less than available income, or (iii) more than available income, the excess purchases being financed by money creation or by dishoarding.

(i) *Buyers spend their available income and nothing more.*

In this case

$$(C+I)_{t+1}=Y_t=Y_{t+1}.$$

Incomes from the first and second periods are equal to each other. This denotes equality between savings and investment. Since buyers spend

exactly their available income, their net hoarding is zero. All savings are invested. Equality $I=S$, then, denotes the dynamic stability of national income.

(ii) *Buyers spend more than their available income.*

Hoarding is negative. Inequality $I > S$ points to increasing income. Behavior of income holders is such that total demand is greater than total supply. Excess demand is absorbed when income is stepped up.

(iii) *Buyers spend less than their available income.*

This signalizes deflation or depression. Net hoarding is positive: $S > I$. The unspent fraction of available income thins out new production. We shall see how this works out.

Consumption and investment expenditures for period t determine the income of this period

$$C_t + I_t \rightarrow Y_t.$$

Income produced during t becomes available in $t+1$.

$$Y_t^{\text{produced}} \rightarrow Y_{t+1}^{\text{available}}$$

Income available in $t+1$ is divided between consumption and investment expenditures and hoarding.

$$Y_{t+1}^{\text{available}} = D_{t+1} + H_{t+1}$$

Expenditures for period $t+1$ determine the income for this period.

$$D_{t+1} \rightarrow Y_{t+1}^{\text{produced}}$$

And, since $D_{t+1} < Y_{t+1}^{\text{available}}$,

$$Y_{t+1}^{\text{produced}} < Y_t^{\text{produced}}.$$

Income produced during $t+1$ is less than the income produced in t.

This drop in the level of national income is explained entirely by the behavior of income holders, who decide to spend only part of their available income.

Dennis ROBERTSON's analysis thus allows the comparative factors of Supply and Demand to determine the level of national income. Demand represents the active force which Supply follows passively. Income is stabilized at a constant level when holders of available income (homes, businesses, state) push their spending to the exact limit of their current incomes. Income increases when holders spend more than their available income, $I > S$. And Income decreases as soon as a fraction of available income is withheld from purchases to be set aside as hoardings, $S > I$.

KEYNES was one of the first to criticize ROBERTSON. He finds fault in two respects with the 'step by step' analysis.

2. "Furthermore, a man cannot know how much he has 'saved' during a given period in Mr. ROBERTSON's sense unless he knows how long Mr. ROBERTSON's technical 'day' is... Thus there is no means by which any of us can tell how much we 'saved' last year, until Mr. ROBERTSON vouchsafes to tell us the length of his technical 'day.' So I think that for the plain man, and still more for any practical purpose, Mr. ROBERTSON's definition is, if possible, even worse than mine."[15]

The Keynesian definition, as we know, does not recognize inequalities $I > S$ or $I < S$. ROBERTSON reacts against identity $I \equiv S$, since it leaves no place for the behavior of buyers. To demolish this identity, ROBERTSON introduces his dynamic analysis, based on a series of so-called significant intervals. These periods, 'Robertsonian days,' are of arbitrary length. But they are vital in differentiating between produced and available incomes, and consequently between Supply and Demand. Only at this price can the behavior of buyers affect the level of national income. Now ROBERTSON wants above all to safeguard the *causal* character of macroeconomic analysis. So it appears that variation in national income is ascribed to the behavior of income holders. If the latter spend all their income and no more, national income remains constant in time. But if demands become greater than the total amount of available income, national income increases. It decreases when available income is not entirely spent in purchases.

Robertsonian days seemingly make causal analysis possible. Even if the real length of the chosen periods is not exactly known or if it is purely conventional, these periods must be introduced into the body of analysis to preserve its causal character. ROBERTSON's answer must be understood in that way. "... I am not insensible to the force of the objections which have been made on the score of the vagueness of the length of the necessary minimum period of lag ('day') assumed.... In any case *some* conception of a lag still seems to me necessary to protect us against the peril of confounding causes with results..."[16]

Is KEYNES' second criticism more conclusive?

"Even if a slump were to last for years and incomes were to fall to half their previous level, the total excess of Saving over Investment for the whole period could not in the aggregate exceed half a week's (day's) income. Thus... the difference between Investment and his (Mr. ROBERTSON's) 'Saving' can never be large enough to have practical importance, unless his 'day' is a substantial period."[17]

Since ROBERTSON's analysis covers not only depressions but the total cycle, KEYNES' objection is not pertinent. If income actually decreases by half and then resumes its initial level, the cumulative difference between savings and investment is zero. However, the successive positive and negative differences between S and I explain the variation of national income from day to day.

'days'	0	1	2	3	4	5	6
produced income	200	150	100	100	150	200	
available income		200	150	100	100	150	200

The difference between the sum of available incomes and the sum of produced incomes is zero, or $S - I = 0$. Yet savings are greater than investment on days 1 and 2, and investment exceeds saving on days 4 and 5. These differences are 'Robertsonian causes' of variation in level Y, which drops from 200 down to 150 (first day's hoarding), then to 100 (second day's hoarding). After a day's stabilization, level Y rises from 100 to 150 (fourth day's negative hoarding) and from 150 to 200 (fifth day's dishoarding).

KEYNES' basic criticism of 'step by step' analysis is found in his *General Theory*. But before quoting KEYNES, we can perhaps discover for ourselves the weakness in ROBERTSON's logic.

3. ROBERTSON is surely free to assume that income is not immediately available:

$$Y_t^{\text{produced}} \rightarrow Y_{t+1}^{\text{available}}$$

How could income be spent in the very transaction which produces it? The lag seems necessary. First (period t), income is produced, and it is spent consecutively (period $t+1$).

ROBERTSON's whole argument is based on the premise that hoarding can be positive or negative. If it had to be zero, $H \equiv 0$, causal analysis would completely fail, for the *behavior of buyers would no longer explain* the difference between income of period $t+1$ and income of period t. Now hoarding is really logically nil. ROBERTSON starts from KEYNES' definition of national income, $Y \equiv C+I$. He alters it slightly in order to authorize the positive and negative hoarding which he judges vital for causal analysis. In other words, ROBERTSON accepts KEYNES' definition,[18] but with a proviso that the two terms of equation $Y=C+I$ should be attributed to two different periods. Thus Y belongs to period t while $C+I$ occurs in period $t+1$.

CONNECTIONS BETWEEN INCOME AND DEMAND

Keynesian analysis	Robertsonian analysis
$(C+I)_t$ and Y_t	$(C+I)_t$ and Y_{t-1}
$(C+I)_{t+1}$ and Y_{t+1}	$(C+I)_{t+1}$ and Y_t

In ROBERTSON's analysis, income spent in period t for any purchases, $(C+I)_t$, is the income of the preceding period, Y_{t-1}. In KEYNES' analysis, income spent on the output of period t, $(C+I)_t$, defines the income of this same period, Y_t. To avoid any mistaken interpretations, we must remember that national income is, by definition and *in both analyses*, equal to the corresponding expenditures in purchases.

Keynesian analysis

$$Y_t \equiv (C+I)_t$$

$$Y_{t+1} \equiv (C+I)_{t+1}$$

Robertsonian analysis

$$Y_{t-1} \equiv (C+I)_t$$

$$Y_t \equiv (C+I)_{t+1}$$

Even in ROBERTSON's analysis, Income has to equal Demand. Step by step analysis simply shifts the second term of the identity, delaying it by one period, 'one day,' with respect to the first term. Therefore macroeconomic hoarding is necessarily nil even in ROBERTSON's analysis.

$$Y_{t+1}^{\text{available}} \equiv D_{t+1} + H_{t+1}$$

where $H_{t+1} \equiv 0$. ROBERTSON's causal analysis is thus completely voided, for macroeconomic purchases must always be exactly equal to total available incomes, whatever the behavior of buyers.

ROBERTSON's analysis seems less original after a second look at KEYNES' analysis. The link between Income and Demand should be sought with respect to a logical rather than a chronological criterion. Identity $Y_t=(C+I)_t$ does not mean that output and demand occur in the same period. Quantities Y_t and $(C+I)_t$ refer to the output of the same period t, even if demand should occur subsequently.

So the Robertsonian analysis shows up as a special case of KEYNES'.

We must now examine the last sequence of the Robertsonian analysis:

$$D_{t+1} \rightarrow Y_{t+1}^{\text{produced}}.$$

This statement is erroneous. Since hoarding cannot be positive or negative even in ROBERTSON's analysis, expenditures are necessarily equal to available income Y_{t+1}, itself equal to produced income Y_t. The sequence $D_{t+1} \rightarrow Y_{t+1}^{\text{produced}}$ would yield a constant income in time. We can only avoid this undesirable result by severing the connection between expenditure of income produced in period t and production of the new income in period $t+1$.

Careful examination of ROBERTSON's dynamic analysis proves, then, that expenditure D_{t+1} is the other side of the Income produced in period t. In short, D_{t+1} corresponds to produced income Y_t and not to produced income Y_{t+1}. The logical sequence is the following:

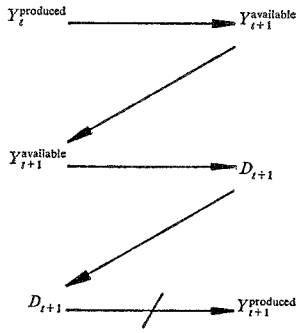


Fig. 16

The chain is broken between D_{t+1} *and* $Y_{t+1}^{\text{produced}}$. *The income produced in any period cannot result from expenditure made from a former income. Determination of income* $Y_{t+1}^{\text{produced}}$ *results from the expected demand relative to* $Y_{t+1}^{\text{produced}}$ *and not from the demand which is the expenditure of* Y_t^{produced}.

ROBERTSON's analysis thus ends in the simple juxtaposition of incomes produced from day to day, with no apparent causal tie between them. This causal tie is not to be found in the 'Robertsonian days' but in the 'effective demand' of KEYNES. "Thus when Mr. ROBERTSON says that there is an excess of saving over investment, he means literally the same thing as I mean when I say that income is falling, and the excess of saving in his sense is exactly equal to the decline of income in my sense. If it were true that current expectations were always determined by yesterday's realised results, to-day's effective demand would be equal to yesterday's income. Thus Mr. ROBERTSON's method might be regarded as an alternative attempt to mine (being, perhaps, a first approximation to it) to make the same distinction, so vital for causal analysis, that I have tried to make by the contrast between effective demand and income."[19]

CHAPTER II

VIRTUAL FACTORS OF TOTAL SUPPLY AND DEMAND

A. POSITIVE ANALYSIS

We will show in this second chapter, as a logical sequence to the first, that, since income is not amenable to dynamic analysis, statics can be dynamized for price but not for income. *Why is this?*

Unless we find the answer to this question, we shall continue to make the mistake signalized in Chapter I. We must distinguish carefully between the two static analyses, price and income. The distinction concerns the factors determining equilibrium. In statics, determining factors are real for prices and virtual for income.

1. Static analysis determines the equilibrium of virtual price. Analogously, static analysis determines equilibrium of virtual income.

2. Static price analysis can be dynamized. No difficulty arises here, for the relevant factors of supply and demand are real in both fields.

3. Static analysis of national income cannot be dynamized, since the determining force is imaginary or virtual and does not exist in real time. The force of adjustment between total demand and total supply cannot be studied in a sequential process. Income must be entirely redefined at each generation; it has no predecessor. By its very nature, the Keynesian analysis of the determination of national income must be exclusively static. In the theory of income, statics is not a special case of dynamics.

4. An excellent criterion is operative in judging theories of national income. If the consumption function is included as a determining factor, the theory cannot be correct. Income must be determined by static factors *before* the consumption function intervenes.

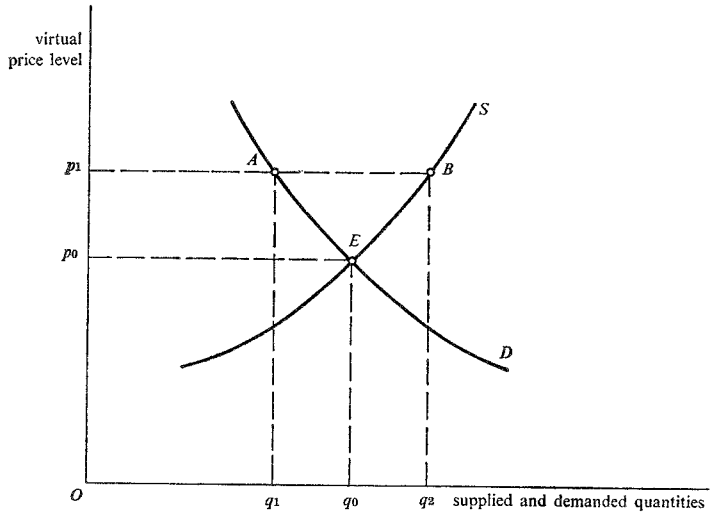
THE TWO STATIC ANALYSES

5. Our purpose is to determine the equilibrium price of a commodity and the equilibrium income of the economy for a single period.

6. These two problems do not concern dynamic equilibrium of the variables, price and income. The equilibrium values we are looking for correspond to a single realized price and a single realized income.

7. In price analysis, any realized price denotes an actual exchange, a two-sided transaction, supply-demand. Price, before being realized, is only virtual. With the help of a chart, we can see exactly how virtual prices must be understood.

Price p_0q_0 takes in both sides of any realized price, supply and demand. On the other hand, price p_1q_1 is only virtual, a demand price but not a supply price. Likewise, price p_1q_2 is merely virtual, a supply price but not a demand price. Therefore, any price which is merely virtual includes only one side of realized price, demand or supply. Conversely, any price not combining supply and demand is a virtual price only. Finally, equilibrium of virtual price is defined when the two virtual prices of supply and demand are integrated. Realized price is therefore equal to the equilibrium value of virtual price.



PRICE DETERMINATION
IN A SINGLE MARKET SESSION

Fig. 17

Realized price is p_0q_0. It is sometimes said that two quantities are determined at the point of intersection between the two curves: on the one hand, the quantities exchanged, x-axis, and on the other, the price level at which the transaction is carried out, y-axis. But the only quantity

determined by interaction of supply and demand is represented on the graph by rectangle p_0Eq_0O. Rectangles p_1Aq_1O and p_1Bq_2O represent virtual prices, demand price and supply price respectively.

8. Static analysis of determination of national income follows the method of static price analysis. But it is useless to start from the *level* of income. We measure *total income* directly on the horizontal axis. And on the vertical axis, we measure total expected demand, represented by OA. From point A we draw the parallel to the x-axis, which intersects the 45° line at E. This gives the solution: Y_0 is the equilibrium of virtual income, so that income realized in the period will be $Y = Y_0$.

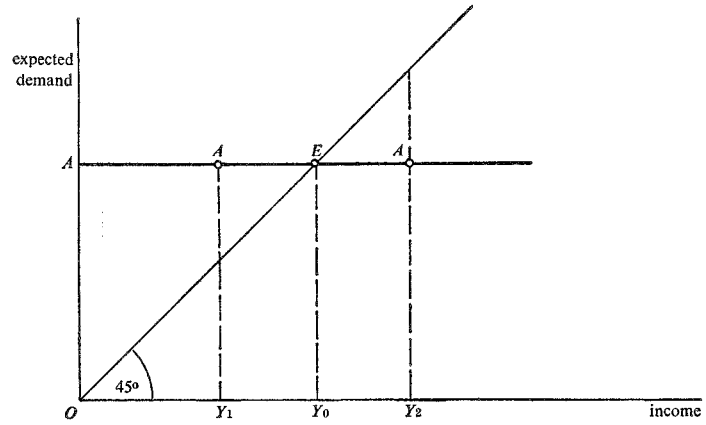


Fig. 18

Income Y has the two sides of any realized income, supply, OY_0, and demand, OA. On the other hand, income Y_1 is purely virtual since it takes in only one side of income. Likewise, income Y_2 is not a supply-demand but a pure supply, the corresponding demand being a separate and smaller quantity: $Y_2A < OY_2$.

Up to this point, reasoning by analogy is helpful. Equilibrium of virtual income is determined by the same method as the equilibrium value of virtual price. However, we know for a fact (Chapter I) that, even if statics can be paralleled in the two areas of price and income determination, the analogy cannot possibly be extended to dynamic analysis.

Why does this analogy suddenly fold up? We must find a clear and decisive answer to this question.

9. Price theory.

Two distinct variables are set up on the horizontal axis: time, and, as before, supply and demand. By dividing the axis into a series of discrete periods, say daily market sessions, the exchangeable quantities of the commodity can be measured inside each period.

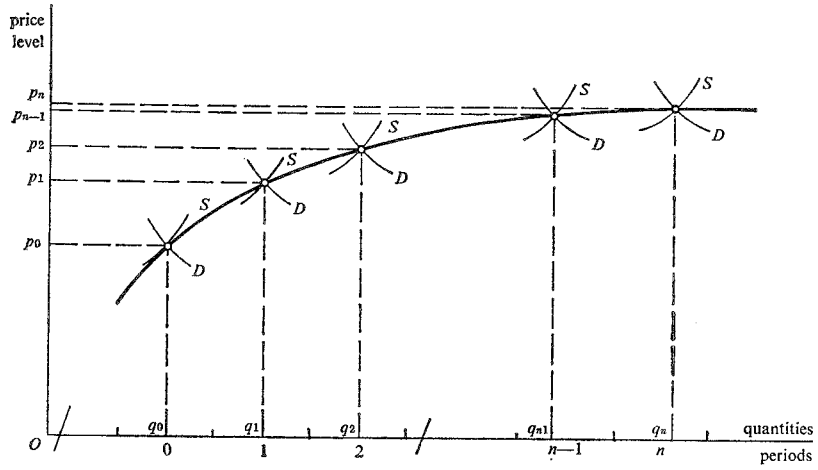


Fig. 19

We shall consider only prices per physical unit of the exchangeable commodity. Unit prices seek their equilibrium inside each market session. In the first session, equilibrium level of price is p_0; in the second p_1, etc.

But realized unit prices also follow another criterion of equilibrium, as appears on the graph. After period n, the unit price of the commodity no longer increases in time. Price level p_n is stable. Every price level in the series $p_0, p_1 \ldots$, defines the equilibrium value of a virtual price. But in this series, price level p_n is also an equilibrium value in another sense, for it defines the stable value of price level in time.

The distinction between the two equilibriums is very clear. In the series $p_0, p_1, p_2, p_3, \ldots, p_n$, all prices are equilibrium prices in the first sense, for they all represent actual sales. But, in this series, only price p_n remains stable *in time*. The level of prices increases on successive markets, $p_n > p_2 > p_1 > p_0$, up to price p_n, where it stabilizes.

The terminology is ambiguous if we speak only of the equilibrium of price level. It would be better to use two terms.

Equilibrium of virtual prices. All realized prices are equilibrium values in this sense, where equilibrium means realized equality between supply and demand.

Equilibrium of realized prices. In the series, realized price levels can increase or decrease so that, in the short-run for example, the price level may tend to stabilize. On the graph, p_n represents the dynamic equilibrium of price level.

The distinction is consistent but not synonymous with the dichotomy between statics and dynamics.

Static analysis. Inside a single market session, supply and demand curves may intersect. Their common point determines the equilibrium of virtual price in the series of all virtual prices.

Dynamic analysis. Here we no longer look for the point of intersection between supply and demand curves. We shift both curves in time.[20] It is improbable that the position of the curves be continuously known in time. But for the sake of dynamic analysis it is enough to be able to spot-check the new positions of the curves on the axes. If successive positions of the curves are such that their points of intersection become stable at level p_n, price level p_n is the dynamic equilibrium of realized price levels.

Dynamic analysis is obviously more interesting than static analysis, which is merely a special case of it. If it is possible to shift curves in time, we know both (i) the new position of the curves in each particular period, information which could be acquired even from static analysis, and (ii) the law governing the change of this position in time, information which can only be acquired in dynamic analysis. Dynamic analysis studies the *concatenation* which realized prices form in real time.

10. *Income theory. Realized incomes do not form a concatenation in time. Dynamic analysis of national income is therefore logically impossible.*

The explanation of this, simple in itself, is difficult to understand because of the influence of the dominant theory.

The determining factor of national income is total demand. This is the first difference relative to price theory, where supply as well as demand is a determining factor. In the theory of income, total supply follows total demand, which is given by the state of expectations. But in both cases, supply and demand adjust to each other, and, methodologically, it makes little difference whether this adjustment be reciprocal or not.

But another and more serious difficulty arises here. Total demand, as realized in time, can be of no help. Determination is possible only if Demand is logically known before Income, which must adjust to it.

We must keep in mind that realized demand and realized income are only head and tail of the same coin (Chapter I). It follows that total realized demand can only be known simultaneously with realized income. To require that national income adjust to dynamic variations in total demand is to ask that national income adjust to itself.

Clearly, then, realized demand is not a determining variable, as it is in static analysis.

We thus arrive at the unavoidable conclusion. As it is used in statics to determine national income, total demand is not a realized factor. The only relevant factor is *imaginary* or *virtual demand.*

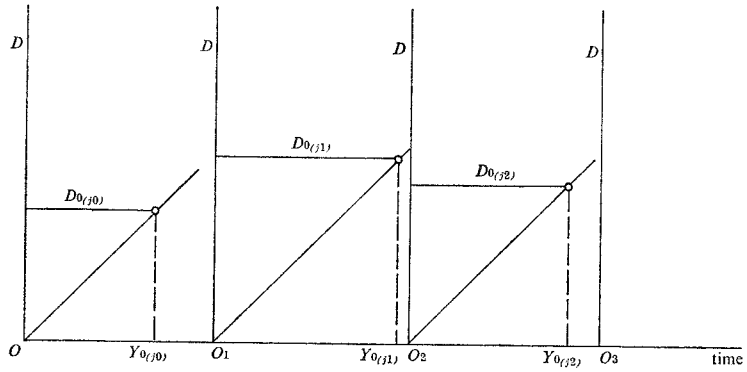


Fig. 20

The vertical axis is shifted in time, from day to day.

Each day's income is determined by adjustment of production to expected demand. According to the firms' estimates, $D_{0(j0)}$ is the demand for J_0's output. Suppose that this demand is measurable in wage units. For example, $D_{0(j0)}$ is the demand for the output of 100 workers employed for a day. Firms employ 100 people the first day. If fewer were employed the total expected demand could not be met. If more were employed, part of the output could not be sold, again according to estimates. Output adjusts to the amount of expected total demand. Income $Y = D_{0(j0)} = = Y_{0(j0)}$ will be produced in day J_0, because $D_{0(j0)}$ is the expected demand for that day's output.

The following days' incomes are determined according to the same procedure. Thus, the second day's equilibrium income is $Y = D_{0(j1)}$, where $D_{0(j1)}$ is the expected demand for the second day's production.

Now, we can try a very simple experiment. Shift line $D_{0(j0)}$ in time, according to the dynamic behavior of demanders. Will this line coincide

with line $D_{0(j_1)}$, and, one day later, with line $D_{0(j_2)}$? *Not at all; lines* $D_{0(j_2)}$ *and* $D_{0(j_1)}$ *are not the new positions of line* $D_{0(j_0)}$ *on days* J_2 *and* J_1.

The different lines D would form a logical sequence in time if the demands which they represent were realized quantities. This is a condition *sine qua non*, for a quantity which does not exist cannot be shifted in time. To say that a quantity is realized is equivalent to saying that it exists in time. Conversely, a purely virtual quantity does not exist in real time. Now, demands $D_{0(j_0)}$, $D_{0(j_1)}$... are expected and virtual quantities, and not realized quantities.

Thus $D_{0(j_0)}$ is the expected demand of the first day's output. But what is the value of the expected demand in J_1? It must be defined in terms of the new output of period J_1. *It leads to line* $D_{0(j_1)}$ *which has no connection with line* $D_{0(j_0)}$.

Expected demands are not real factors, like realized demands, which could be shifted in time according to buyers' behavior. Virtual demands refer to separate periods. They cannot be shifted from period to period. No bridge whatever could be found between the expected demands of outputs belonging to successive periods.

The dictionary defines virtual as "potential, although not in actual fact." Any quantity, then, can be said to be virtual if it lacks some qualities necessary for its realization. With the help of this simple definition, we propose to study five quantities: price, supply and demand of a commodity, national income and total demand. We shall classify them as virtual or realized.

Price and national income are two-sided variables. Price is both a supply and a demand. Income also has the two sides, a supply Y and a demand $C+I$.

Realized price, or purchase, is a single transaction which can be looked at from the two viewpoints of supply and demand ($s \equiv d$).

Virtual price has only one of the two sides necessary to transaction. Virtual price is synonymous with either a supply price or a demand price. On the graph p. 60, price p_1q_1 is only virtual, for at price level p_1, demand is equal to p_1q_1, but supply is different. Likewise, price p_1q_2 is merely virtual, for at price level p_1 supply equals p_1q_2 but the corresponding demand is different. Virtual price is defined by the non-identity $s \not\equiv d$.

Realized income is by definition simultaneously a Supply, Y, and a Demand, $C+I$.

If virtual income has meaning it must be defined $Y \not\equiv C+I$. Virtual income is shown on the chart p. 61. Expected demand, OA, is not a realized income, for it is a pure demand, whereas any realized income is

at the same time supply and demand. But when income has the two sides, Demand, $D=OA$, and Supply, $Y=OY_0$, it is realized.

Consider now the other three variables, supply and demand of a commodity, and total Demand.

A supply which is not simultaneously a demand and a demand which is not a supply define virtual prices. But here we should go beyond the classification of prices as virtual and realized. Demand and supply and not price must now be classified according to the two categories. *Demand and supply of a goods are real even at virtual prices. All demands and all offers which enter into the determination of the microeconomic price of goods belong to the category of realized quantities.*

If we again consult the graph p. 60, we see that price level p_1 is only virtual, since no exchange can take place at this price. Price p_1 still determines behavior of seller and buyer. Thus, demand p_1q_1 is real, for the buyer actually wants to acquire quantity q_1. For the same reason, supply p_1q_2 is real; the seller really offers quantity q_2. Again, prices p_1q_1 and p_1q_2 are virtual. But now we are not concerned with prices, but with supply and demand themselves. Supply p_1q_2 and demand p_1q_1 represent the real and observable behavior of seller and buyer when the unit price of the commodity equals p_1. Price p_1 is virtual because purchasers' and sellers' desires are not mutually consistent at this price. We should not infer that behaviors are themselves virtual. Only the agreement of behaviors is purely virtual at price p_1. As a result price remains virtual. But behaviors are real on both sides.

In income theory, does total demand also represent real behavior? Demand is actually realized when Income is. National income is realized when Supply and Demand are identical. For this reason, Income cannot conceivably be realized unless Demand is also. Conversely, if income is not (yet) realized, demand cannot be either. To virtual income corresponds virtual demand. This is the crucial distinction between the two analyses. *To virtual prices correspond real factors of supply and demand. To virtual income corresponds a total demand which is as virtual as income itself.*

To summarize the contradistinction as clearly as possible: (i) realized Income is connected with a realized Demand, since Income cannot be realized in Supply without being realized also in Demand. Up to this point the theories of income and price are parallel. To realized price corresponds an already realized supply and demand. The proof of this is the same as for income. Price cannot possibly be realized if its two complementary aspects, supply and demand, are not. (ii) To virtual income corresponds a total *virtual* demand. The proof here is somewhat longer.

1° Realized income does not dichotomize supply and demand.

2° Virtual income includes either supply or demand, but not both together. When income is a supply and not supply-demand, it does not fit the definition $Y \equiv C+I$. In short, a supply Y which is not at the same time a demand $C+I$ is virtual in the dictionary sense.

3° Total demand is also a virtual income in so far as it is not identically a demand and a supply. Clearly, to pure supply, $Y \not\equiv C+I$, corresponds pure demand, $C+I \not\equiv Y$. We know therefore that demand is a virtual income when and only when the corresponding supply defines a virtual income.

4° But we have not yet proved the proposition. We have established thus far simply that pure supply and pure demand are virtual incomes. Leaving aside the virtual character of income, which we already have since $S \not\equiv D$, we must now turn to the virtual character of total demand. Income is virtual, but the burden of proof lies with demand and supply.

5° Pure supply, $S \not\equiv D$, already a virtual income, is also a *virtual supply*. If a supply of income were *realized* by a firm, income would be likewise. A real supply could not conceivably be a virtual income. To virtual income must correspond a supply which is itself virtual.

6° Supply being virtual, the corresponding demand is also virtual. Suppose we have a realized total demand, an observable quantity. Then according to its definition, income must also be observable. If demand were realized income would be also. Thus it is clear that a virtual income commands a virtual demand.

Herein lies the basic difference between the theories of price and income.

In price theory, supply and demand always denote real and observable behaviors.

Realized price: supply $\equiv$ demand
Virtual price: supply $\not\equiv$ demand

In the *two* definitions, for virtual price as well as for realized price, supply and demand are *real* factors, the real and observable behaviors of buyer and seller.

But in the theory of national income, *supply and demand can be virtual quantities.*

Realized income: supply $\equiv$ demand
Virtual income: { pure supply, $S \not\equiv D$ / pure demand, $D \not\equiv S$

In these two definitions, supply and demand are ambiguous terms. For realized income, supply and demand are realized quantities. *And for virtual income, supply and demand are virtual quantities.*

*
* *

Here lies the dividing line between price and income and also between neoclassical and Keynesian economics.

Theory of price determination

Pure supply price is not a realized price, but a virtual price. Nevertheless it is not a virtual supply, but a real supply. On the graph p. 60, the supplier offers quantity q_2 of the commodity at unit price p_1. Purchase p_1q_2 does not take place, since the consumer only wants quantity q_1 of the commodity at unit price p_1. Pure demand is also real. The consumer really wants to buy quantity q_1 of the commodity at unit price p_1.

We conclude, then, that pure supply and pure demand are the *real* factors of price determination. Pure demand is a virtual price but a real demand; pure supply is a real supply although a virtual price.

Theory of income determination

On the graph p. 61, demand $D=OA$ is expected with respect to a single period's output. Pure Demand in contradistinction to Demand-Supply denotes a virtual income. But pure Demand is not only virtual Income, it also defines a *virtual Demand.* If Demand were realized, the corresponding Income would be realized.

The determining factor in national income, pure Demand, is virtual. National Income is not determined by realized Demand. Once demand is realized, income is already determined, since no adjustment is possible between two identical terms. Virtual Income is brought into equilibrium with virtual, expected Demand. In Income theory, the determining factor is virtual.

*
* *

Suppose that, in March, we want to look ahead to May. What production shall we undertake in May, if we are bound by no already existing labor contracts?[21]

Firms will hire employees whose production in goods and services will just equal the total expected demand. If for May's output D_0 is the value of this expected demand, then the equilibrium income is

$$Y = Y_{0\,(\text{May output})} = D_{0\,(\text{May output})}.$$

Supply Y adjusts to expected demand D_0.

What exactly is demand D_0 for May's output?

1° It is pure Demand and not Demand-Supply. Firms, expecting a demand D_0, adjust the level of their production and employment to this expected demand. Virtual Demand can meaningfully be taken for an excess Demand.

2° It follows that income is free to vary so as to adjust to $D_{0\,(\text{May's output})}$. The adjustable income is virtual; it is a pure Supply.

3° But if demand $D_{0(\text{May's output})}$ is, first, a purely virtual income, it is likewise a virtual demand. Before May's output is actually produced, the corresponding demand—of goods not yet produced—must be a purely virtual quantity.

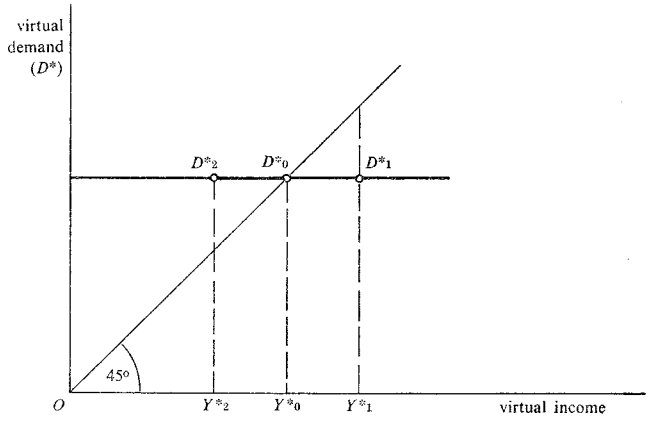


Fig. 21

We have introduced symbols D^* and Y^*. The asterisks distinguish virtual from realized quantities, a distinction of basic importance in macroeconomics.

Compare D^*_2 and Y^*_2.

If we were to write D_2 and Y_2 instead of D^*_2 and Y^*_2, the letters would stand for total demand and national income for May's output. As a result, we would again be burdened with identity $S \equiv D$. But the graph represents virtual quantities D^*_2 and Y^*_2, since the determining factor of national income is virtual. Demand D^*_2 is not realized, for the corresponding income Y^*_2 will never be produced; income actually produced in the period is once and for all different from Y^*_2.

Now compare D^*_1 and Y^*_1.

As the two quantities are unequal, neither one is realized. Income Y^*_1 is larger than demand D^*_1. The result is that firms will not produce in May an income equal to Y^*_1, but a smaller one. They aim to produce only goods which will be demanded. Income Y^*_1 is then conclusively virtual. It will never be realized, for firms will give it up definitively, at least for May's output.

To conclude paragraph 10, which shows the logical impossibility of the dynamic determination of national income, we can refer to the graph p. 61. Defined for periods J_0, J_1, J_2, demands $D_{0(J_0)}$, $D_{0(J_1)}$, $D_{0(J_2)}$, ..., are virtual factors. To avoid confusion, stars should be added, $D^*_{0(J_0)}$, etc. Analysis of national income determination is necessarily static for the reason stated above: in each period income is adjusted to virtual demand, a quantity which, contrary to realized demand, does not exist in real time. How could a quantity not located in time be followed dynamically?

In *price* theory, even pure demand and pure supply are real factors which can be followed in time.

11. The crux of the matter is explicable in terms of demand functions.

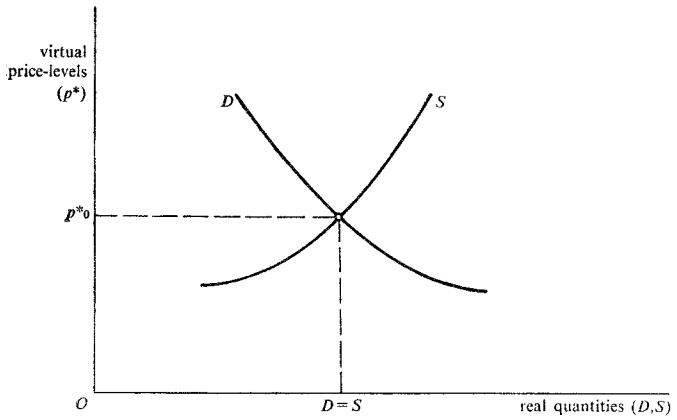


Fig. 22

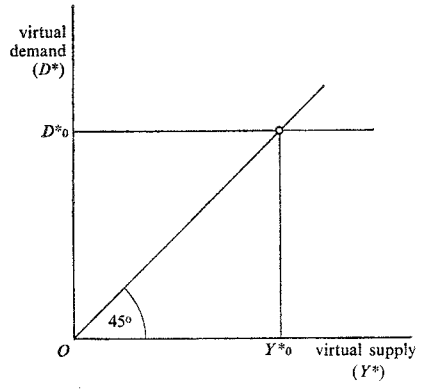


Fig. 23

Compare D and D^*.

All points on curve D are defined by function $D = D(p^*)$.

A single point on line D^*, point D^*_0, is defined by function $D = D(Y)$. Other points on line D^* obey function $D^* = D^*(Y^*)$.

In statical price theory, demand as well as supply is a real factor as a function of virtual prices. In (statical) income theory, real Demand is known only as a function of realized Income, that is, only at equilibrium point D^*_0. All other values relate virtual Demand to virtual Income. Function $D = D(Y)$ is therefore totally inadequate in determining Y.

As regards price theory, the determining factors are real supply and real demand. When it comes to Income theory, the determining factor is virtual Demand.

(i) The function of real Demand is

$$D = D(Y) = Y.$$

(ii) The function of virtual Demand is

$$D^* = D^*(Y^*) = D^*_0.$$

Whatever the value of Y^*, the corresponding value of virtual Demand is D^*_0. The total expected demand of the period's output is given in a single figure, D^*_0.

Failure to distinguish between the two functions of total Demand is responsible for many fundamental mistakes in the dominant theory. The principal error is doubtless the use of the consumption function for determining national income.

THE CONSUMPTION FUNCTION GOVERNS THE DIVISION OF A GIVEN INCOME BETWEEN CONSUMPTION AND INVESTMENT EXPENDITURES. IT CONTRIBUTES IN NO WAY TO NATIONAL INCOME DETERMINATION

12. If total consumption is really a function of national income, we have $C=C(Y)$. The argument is not affected by the form of the function. We can write

$$C_t=C(t),$$

or $$C_t=C(Y_{t-1}),$$

or again $$C_t=C(Y_{\text{permanent}}),$$

and finally $$C_t=C(Y_0,\ Y_1,\ldots,\ Y_{t-2},\ Y_{t-1}).$$

In all these cases, and in all others which can be devised, the consumption function *has no part in the determination of national income.*

The logical order is the following:

1° Determination of income from virtual demand — virtual demand function
2° Realized income
3° Expenditure of national income in consumption (and investment) — function of real consumption

The function of realized consumption intervenes after income is already realized. *Virtual* demand is the only determining factor. For example, with function $C=\frac{1}{2}Y$, C is determined when we know Y. But conversely the consumption function gives no information about the value of income. In brief, *determination of* Y takes place through the function of virtual demand; *determination of* C is the result of the consumption function. It is clear that if we knew only function $C=\frac{1}{2}Y$, we would know exactly nothing about the value of national income, consumption being equal to half of *any* income.

Again,

—Knowledge of consumption as a function of income is an insufficient condition for the determination of national income. This is recognized by the dominant theory.

—Knowledge of consumption as a function of national income is not a necessary condition for income determination. It is useful only for

dividing a given amount of income between consumption and investment. This fact is misinterpreted by the dominant theory. Whatever the buyers' propensity to consume or to save, it has no influence on the size of national income.

13. We shall now assume that the total function $D^* = D^*(Y^*)$ is divided into two partial functions of virtual consumption and investment.

We can see that the two functions are not complementary in Y^*. Complementarity would imply that for any value of virtual income, Y^* would be known before its components C^* and I^*. In other words, equation $Y^* = C^* + I^*$ would be true for any value of virtual income, and of course it is not. In fact, Y^* equals $C^* + I^*$ only for Y^*_0, the equilibrium value of virtual income. Therefore consumption may vary while investment is maintained constant:

(1) $$C^* = C^*(Y^*) = a + bY^*, \text{ for example,}$$

(2) $$I^* = I^*_0.$$

These two equations are central in the dominant theory. To determine Y, combine them with the definition of equilibrium income:

(3) $$Y^*_0 = C^* + I^*.$$

The result is SAMUELSON's "seminal" equation, here in linear form.

$$Y^*_0 = C^* + I^* = a + bY^*_0 + I^*_0$$

In the field of realized quantities, it is easy to criticize the dominant theory, as we have repeatedly shown. Realized income being necessarily equal to corresponding expenditures of consumption and investment, Y cannot be determined by $C + I$. But here we are confronted by an entirely new problem. Now the quantities are virtual and virtual income is equal to virtual consumption and investment expenditures only for its equilibrium value Y^*_0.

Function $C^* = C^*(Y^*)$ has no proper meaning. Its apparent meaning comes from function $C = C(Y)$ with which the dominant theory confuses it.

If we could speak of a virtual consumption function, it would have to be written $C^* = C^*(Y^{*c})$ and not $C^* = C^*(Y^*)$. The function of realized consumption must be defined in terms of *total* income. If we can demonstrate that the virtual consumption function has no possible meaning except in terms of *partial* income, Y^{*c}, it will be clear that the consumption function contributes nothing to the determination of national income.

Suppose that there exists a function $C^* = C^*(Y^*)$. In this case, any value of variable Y^* is purely virtual, except the equilibrium value for which $Y^* = D^* = C^* + I^*$.

The dominant theory does not consider the possibility of a function $C^* = C^*(Y^*)$. However, if the question of such a function arises, it is because theory has failed to distinguish between virtual and realized quantities. In other words, function $C^* = C^*(Y^*)$ carries over function $C = C(Y)$ from the area of realized quantities into that of virtual quantities. The theorist must show the logical error in this procedure.

Examine determination of national income in the sector producing consumer' goods.

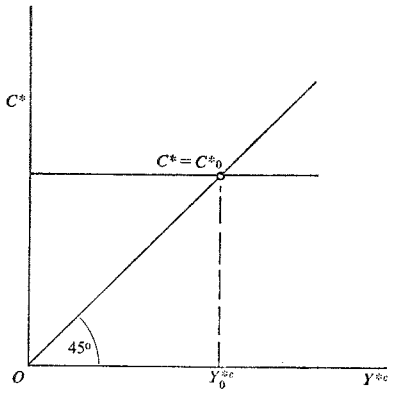


Fig. 24

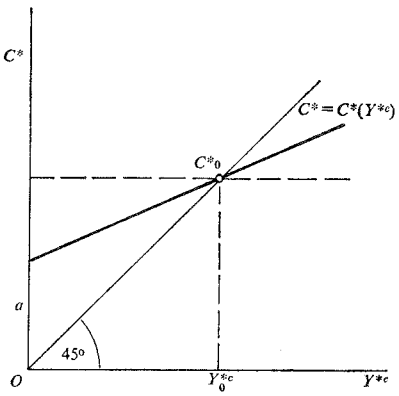


Fig. 25

Figure (24) illustrates function $C^* = C^*(Y^*) = C^*_0$ or $C^* = C^*(Y^{*c}) = = C^*_0$. For any value of virtual income, total or partial, the corresponding virtual consumption is the constant C^*_0.

Figure (25) represents the function of the assumed type, $C^* = C^*(Y^*) = = a + bY^*$. In expression

$$bY^*,$$

is Y^* total income or is it income produced in the sector of consumer goods only?

We shall assume that we know a, which can represent any constant, although no one knows or ever will know the value of the goods that society must consume to survive. But society consumes not only up to a, for 'superfluous' consumption must be added to necessary consumption, to the amount bY^*. However bY^* is just as inexact as a, for who knows when necessary consumption stops and superfluous consumption begins? The only real question is: should we write bY^* or bY^{*c}? Obviously, the answer is bY^{*c}.

Suppose that virtual equilibrium income increases suddenly in the sector producing investment goods.

Does this increase have a repercussion upon the equilibrium of virtual income in the first sector? Logically it can have none.

Income supply in the second sector, Y^{*i+di}_0, is totally absorbed by the corresponding demand, for the criterion of equilibrium is

$$Y^{*i+di}_0 = I^*_0 + \Delta I^*_0.$$

It follows that no fraction of income supplied in the second sector can be spent in the first.[22]

Therefore the function of virtual consumption is

$$C^* = C^*(Y^{*c})$$

and not $C^* = C^*(Y^*)$.

To conclude this chapter, we briefly sum up the main points.

1° The two functions

$$C = C(Y)$$

$$C^* = C^*(Y^{*c})$$

must be kept separate. In particular, income included in the virtual function is a partial income, whereas income included in the realized function is total income.

2° Which of these two distinct functions determines national income?

Total income is determined by the value of function $D^* = D^*(Y^*)$ which equalizes Y^* and D^*.

Therefore virtual consumption and investment are the only determining factors.

3° Function $C = C(Y)$ can only be experimental. It depends on realized factors. The consumption function intervenes *after* income is entirely determined from virtual factors.

4° It could still be objected that determination of income to be produced in the two sectors of the economy depends on the consumption function. For example, if $C = {}^1/_2 Y$, it would seem that each sector must produce half the national income. If not, the function of realized consumption would be invalidated.

5° It is certain however that the function of realized consumption in no way affects the determination of income, which is entirely derived

from the virtual factor of expected Demand. If in the n^{th} period, each sector does not produce half the national income, the consumption function $C=\frac{1}{2}Y$ is invalidated. Indeed, since the function is simply experimental, it is the function, and not the experiment, which must yield.

6° Therefore we can revert to the simple function

$$C^* = C^*_0.$$

Function $C^* = C^*(Y^*)$ where C^* is variable with virtual income results from the mistaken confusion between virtual and realized quantities.

7° The logical sequence is, finally:

(i) determination of virtual income equilibrium in the two sectors,

$$Y^*_0 = D^*_0,$$

(ii) division of the already determined income between consumption and investment.

The consumption function belongs to the second logical level (ii), whereas determination of national income logically belongs to the preceding level (i).

The two logical steps (i) and (ii) can be distinguished more easily if we do not divide national economy into two sectors. It is well known that the category to which any goods or services belong depends on the identity of the buyer. The same goods are for consumption or investment according to whether they are bought by a firm or by a household. Consequently, adjustment of virtual incomes to expected demands occurs in each particular firm, however small, irrespective of the sector to which it belongs. Determination at level (i) thus simply becomes

$$Y^*_0 = D^*_0,$$

where Y^*_0 is the sum of equilibriums of virtual incomes and D^*_0 the sum of expected demands, with Y^*_0 and D^*_0 specified for the same period's output. Output of the period once known, we can then turn to determination at level (ii), the given income being divided up into savings and consumption, or consumption and investment.

B. CRITICAL APPRAISAL OF THE DOMINANT SCHOOL

We will examine four authors.

Lawrence R. KLEIN, in his excellent doctoral dissertation for M.I.T., *The Keynesian Revolution*, just stops short of drawing the basic distinction

between income theory and price theory. At the threshold he steps back and remains in the Marshallian and neoclassical tradition.

Over a hundred pages of Gardner ACKLEY's *Macroeconomic Theory* discuss the consumption function. This function would not have seemed so important to the author if he had not thought it indispensable in determining national income. We know, however, that logically this function contributes nothing to income determination.

A young scholar, Axel LEIJONHUFVUD, in his remarkable book, *On Keynesian Economics and the Economics of Keynes*,[23] has undertaken to demonstrate that the *General Theory*, such as it is taught everywhere by the dominant School, is inadequate. Did the author find any formal error in the Keynesian income-expenditure model? We believe that LEIJONHUFVUD found no fault with the three-equation system:

$$Y = C + I$$

$$C = C(Y)$$

and $$I = I(Y);\ I = \bar{I} \text{ (special case).}$$

Here he detects no mistake in logic. His idea is that these equations do not trigger off a revolution in thought but only shift the emphasis from price to income theory. Economy, after a disequilibrating shock, will resume equilibrium through either price or income adjustment. Price adjustment agrees with MARSHALL's *Principles*. But the Keynesian system, either simplified or complete,[24] emphasizes income adjustment. This position obviously denies any basic originality in the Keynesian determination of national income. An initial disequilibrium is followed by either a price or a quantity adjustment, if not both together. But KEYNES' originality, as LEIJONHUFVUD has it, lies in his awareness that operators' information must be paid for, a cost which WALRAS in his analysis wrongly gives as zero. "His [KEYNES'] model is characterized by the absence of a 'Walrasian auctioneer' assumed to furnish, without charge and without delay, all the information needed to obtain the perfect coordination of the activities (both spot and future) of all traders." (p. 48) From here on, without respect to any cybernetic consideration, we need only prove that the author gives too much credit to the Keynesian income-expenditure model. To be convincing, he should have demonstrated the lack of logic in the Keynesian dominant theory.

Lastly we shall examine the *General Theory* itself. The principle of effective demand opened up a new field to economic theory, which brings in virtual factors. KEYNES stated the determination of national income clearly in terms of expected Demand. Although, from then on, he had

only to remain faithful to his own innovative thought, he himself caused the confusion between expected Demand and realized Demand. It is not surprising that KEYNES could not help but revert back to the tradition which he had held for so long. "The composition of this book has been for the author a long struggle of escape, and so must the reading of it be for most readers if the author's assault upon them is to be successful, —a struggle of escape from habitual modes of thought and expression." (General Theory, p. viii.)

Today, after a lapse of forty years, it is much easier to oppose KEYNES' originality to Marshallian tradition.

LAWRENCE R. KLEIN

Lawrence KLEIN is aware of the distinction between realized and virtual quantities. His text expressly states it. But once having established the distinction, he then denies its logical consequences. His theory quite naturally carries him over to dynamic analysis of income determination. However, only static analysis of income is logically feasible, for virtual quantities can only be apprehended in statics.

REALIZED QUANTITIES AND VIRTUAL QUANTITIES IN KLEIN'S CONTRIBUTION

"The economic process is viewed as made up of a series of intersection or equilibrium points of savings and investment schedules. The observed level of national income for each time point can be considered as the equilibrium level of income corresponding to a set of savings-investment schedules. The observed levels of savings and investment are those two values on the schedules corresponding to the observed level of income. All the other values of savings and investment along the schedules are not observed; they are virtual levels of savings and investment corresponding to levels of national income other than that level which actually takes place. The virtual levels of savings and investment are not equal." (p. 111)

In this text, KLEIN's purpose was to study the determination of national income and not the 'classical' equalization of savings and investment, which would result only in determining the rate of interest.

The presentation will probably be clearer if we confine ourselves strictly to national income from the two viewpoints of Supply and Demand.

When the line of expected Demand intersects the 45° line, income is at its equilibrium level, Y^*_0. This level defines the value of income to be produced and therefore the value of realized income. And for any realized income the corresponding value of realized savings and investment must be equal to each other. In fact, any difference between savings and investment can always be identically found between total supply and total demand:

$$Y-(C+I)=S-I.$$

In this respect, KLEIN's text cannot be objected to.

In the case of *realized* income, we necessarily have

$$Y-(C+I)=0,$$

and

$$S-I=0.$$

But for any other value of national income, that is, for a value of this income which is not realized, we have

$$Y-(C+I)\neq 0$$

and

$$S-I\neq 0.$$

Using symbols which differentiate between realized quantities and virtual quantities, we have

$$S^*-I^*\not\equiv 0,$$

or in the same way

$$Y^*-(C^*+I^*)\not\equiv 0,$$

although

$$Y\equiv C+I,$$

and

$$S-I\equiv 0.$$

We thus reach the same conclusion as KLEIN. To the right and the left of the equilibrium point, investment and savings are not identical since they are virtual factors. Only at equilibrium, when total supply is equal to total demand, are savings and investment identical; for then they are realized. In other words, the realized national income Y_0 implies the necessary equality of Y and $C+I$, or identically of S and I. But for any non-realized value of national income,

$$Y^* \neq Y^*_0,$$

we can verify the non-identity of Y and $C+I$, and likewise the positive and negative differences between virtual saving and investment.

"The same situation occurs in the familiar theory of supply and demand. As observables over time, supply and demand are always equal in so far as they just represent opposite sides of the same transaction. But as static schedules, supply and demand are related in a genuine equation and are not identical. In the latter instance one can talk about divergences between supply and demand at *virtual, unobserved* prices. Similarly, one can talk about divergences of savings and investment at *virtual, unobserved* levels of income." (pp. 111-112)

For any value of income Y^* different from Y^*_0, we have $S^* \neq I^*$. So we can indeed "talk about divergences of savings and investment at *virtual, unobserved* levels of income."

CONFUSION BETWEEN VIRTUAL AND REALIZED QUANTITIES IN KLEIN'S TEXT

Logical argument

1° When income is virtual, so are savings and investments. "...; they are virtual levels of savings and investment corresponding to levels of national income other than that level which actually takes place." (p. 111)

2° When income is realized, so are savings and investment. "The term 'observable savings' refers to that particular level of savings calculated from the savings schedule from a knowledge of the unique equilibrium value of national income which equates savings and investment. Observable investment is calculated from the investment schedule at the same level of national income. The observable values of savings and investment are single points, while the schedules of savings and investment form continuous series of points along curves." (pp. 110–111)

Inconsistency

1° When income is at its equilibrium level, the difference between savings and investment is necessarily nil. "In equilibrium, income has a zero rate of change; it is neither rising nor falling. The equilibrium, in this sense, implies that there is no difference between savings and investment." (p. 113)

2° When income has not reached its equilibrium level, the difference between saving and investment is either positive or negative. "the rate

of change of income depends upon the difference between savings and investment such that income rises when investment exceeds savings, and income falls when savings exceed investment." (p. 113)

According to logical reasoning, savings and investment are either realized or virtual quantities. They are virtual quantities for any virtual value of national income. In other words, to an unproduced income, different from the production "which actually takes place," corresponds a saving and an investment which are also virtual quantities, that is, literally, quantities which do not exist.

In the fallacious argument, saving and investment are always real quantities. But realized savings and investment are equal or unequal, depending on circumstances. When saving exceeds investment, income decreases. When investment exceeds saving, income increases. When the two factors are equal, national income is stable in time. "The equilibrium, in this sense, implies that there is no difference between savings and investment." (p. 113)

On the one hand (logical argument), savings and investment are necessarily equal to each other. On the other hand (fallacious argument), realized savings and investment can be different quantities pointing to a disequilibrium of realized income. Logically, savings and investment cannot exist if they are unequal. No difference can obtain between realized savings and investment. Illogically, such a difference can obtain, and if it does income increases or decreases until realized savings and investment are again equal.

The logical argument conforms to the theory of income, while the fallacious argument is borrowed from the theory of prices.

Price theory. Even *virtual* price defines *real* factors of demand and supply. If supply and demand are unequal for a given virtual price, price must increase or decrease until supply and demand are equal. This is obviously a case of static analysis. The variable is virtual price, not realized price. The only realized price involved is the equilibrium of virtual price.

Income theory. *Virtual* income defines *virtual* factors of demand and supply. When Supply and Demand are unequal for a given virtual income, it must increase or decrease until virtual Supply and Demand are equalized. The variable is virtual and not realized income. In other words, analysis is concerned with virtual and not with realized income. The only realized income involved is the equilibrium value of virtual income, for which virtual savings and investment, or virtual Supply and expected Demand, are balanced.

Price theory. Any difference, positive or negative, between the supply and demand of a commodity at virtual price levels is a real difference. It is observable in time, for example during a market session.

Income theory. Any difference, positive or negative, between total supply and demand, or between savings and investment, is a virtual difference which does not exist in time, for it shows the gap between expected Demand and an Income which is not actually produced. No difference between macroeconomic savings and investment can ever be observed.

Price theory. Factors of supply and demand, real even in statics, can be logically shifted into dynamic analysis.

Income theory. Adjustment of virtual Income to expected Demand cannot be studied in time. There is no bridge between statics and dynamics.

From this follows the second criticism to be levelled at KLEIN.

REFUTATION OF KLEIN'S DYNAMIC INCOME THEORY

"The idea of regarding any observed value of national income as the equilibrium value corresponding to an equation between savings and investment, in the schedule sense, is somewhat artificial. A more realistic view is that the observed levels of national income are observed as the result of a continuous dynamical process. Dynamical processes have been long constructed to apply to demand and supply in individual markets. A familiar dynamical scheme runs as follows: Supply depends upon the price level; demand depends upon the price level; and the rate of change of price depends upon the difference between supply and demand such that prices go up when demand exceeds supply and prices go down when supply exceeds demand. It is very simple to form a mathematical model of this market structure and determine the exact time paths of each of the variables involved.

"In the above example, price serves as the equilibrating variable which always adjusts so as to bring supply and demand into equilibrium. The analogue of this adjustment process in the Keynesian system can easily be formulated in terms of a dynamical model. In this model, savings depend upon the level of income; investment depends upon the level of income; the rate of change of income depends upon the difference between savings and investment such that income rises when investment exceeds savings, and income falls when savings exceed investment." (pp. 112–113)

This long quotation refers to the central idea in the macroeconomic analysis of income: is it possible to reason in dynamics? KLEIN never really asks the question. He takes dynamics for granted. His only justification lies in the analogy between price and income determination. But the author merely assumes the analogy without bothering to prove it. If KLEIN had recognized the problem, he would have been led to the conclusion that dynamics is not a method suitable to the theory of income determination.

The difference between macroeconomic investment and savings is a function of national income:

$$I - S = f(Y).$$

What is the form of this function? Professor KLEIN is on the right track when he points out two savings and two investments. "The observed levels of savings and investment are those two values on the schedules corresponding to the observed level of income. All the other values of savings and investment along the schedules are not observed; they are virtual levels of savings and investment corresponding to levels of national income other than that level which actually takes place." (p. 111)

Two separate symbols should be used in order to avoid confusion: S^* and I^* for the values of virtual savings and investment; S and I for values of realized savings and investment. Quantities S and I refer to realized income, that is, to the equilibrium point of virtual incomes, Y^*_0. Quantities S^* and I^* relate to any virtual incomes Y^*.

KLEIN succumbs to the only remaining snare, that is, he fails to keep separate the quantities which he himself differentiated in the first place. This mistake we must be careful to avoid.

The two functions we are looking for can be stated in simple form.

$$I - S = f(Y) = 0 \tag{1}$$

Since Y is realized income, corresponding investment and savings are also realized. For any gap to occur between savings and investment, Income would have to be different from the corresponding Demand, an absurd proposition, for by definition $Y \equiv C + I$.

The second function reads

$$I^* - S^* = f(Y^*) = D^*_0 - Y^*. \tag{2}$$

For any value of Y^*, virtual income, the difference between virtual investment and saving is measured by the vertical distance between $D^* = D^*_0$ and the 45° line.

Thus it is clear that distinct quantities predicate distinct functions. Function $I - S = f(Y)$ is of no use in determining national income. If savings are necessarily equal to investment for any realized income, obviously no equalization of I and S can determine *the* equilibrium value (?) of national income. On the contrary, function $I^* - S^* = f(Y^*)$ determines the virtual equilibrium value Y^*_0. Income Y^* is balanced only for value Y^*_0 which equalizes virtual investment and savings. When $I^* - S^*$ equals 0, $D^*_0 - Y^*$ equals 0.

In brief,

(i) Equalization of savings and investment can be used to determine the equilibrium value of national income.

(ii) In dynamics, the relevant variables are *realized* savings and investment, S and I. But S and I are the two sides of the same thing. Therefore, any attempt to determine national income equilibrium through the adjustment of these two *identical* variables must be vain.

(iii) In statics, the relevant variables are *virtual* savings and investment, S^* and I^*. As the only link between variables S^* and I^* and variables S and I is S^*_0 and I^*_0, where $S^*_0 = I^*_0 = S = I$, income determination exclusively belongs to the field of static analysis.

*
* *

The following quotation, with which we conclude our examination of *The Keynesian Revolution*, shows how to avoid the confusion so often encountered.

"The beautiful thing about this article[25] is that the analysis of the *General Theory* was so clearly stated, and that such an important element as the savings-investment equation was presented in terms of the real issues involved instead of in terms of trivial, terminological controversy. The pre-*General Theory* Keynesians were much clearer than their successors! In this discussion Mrs. ROBINSON told explicitly how savings and investment can be equal at all points of time and yet not be identical schedules or curves. She first assumed a disequilibrium and then showed the conceptual process by which the level of real income would adjust to bring savings and investment into equality." (pp. 39-40)

We do have to differentiate two propositions:

1° Savings and investment, as identical variables, are always equal.

2° Savings and investment are quantities which are always equal.

The second proposition does not imply identity of savings and investment.

Identity includes equality but equality *even when constant* does not imply identity. In fact, equality can constantly result from an instantaneous adjustment. This is what Mrs. ROBINSON meant. Differences can appear between saving and investment but they are immediately compensated by variation in the level of national income. The gap $I - S > 0$ ($I - S < 0$) is at once filled by an increase (a decrease) in income. *Income is therefore dynamically subjected to the differences between savings and investment although these differences never appear at any moment in real time.*

However, the fact that we must distinguish between propositions (1°) and (2°) leaves the basic question open. Is equality between savings and investment of the first or the second type? The following generally accepted definitions provide the answer.

$$S \equiv Y - C$$

Savings are the unconsumed part of national income. Investment appears in the definition of national income,

$$Y \equiv C + I,$$

$$I \equiv Y - C.$$

The two definitions taken together give

$$I \equiv S.$$

A priori, equality of savings and investment could be due to one of two causes. (1) The variables adjust to each other by fluctuation of national income. (2) The two variables are identical. Since the second case holds by definition, national income need not vary to make I equal to S. And if income varies anyway, from another cause, the equality of I and S is unaffected.

One point should have worried Mrs. Joan ROBINSON and Professor KLEIN who quotes her with admiration. If equality $I = S$ were due to a mutual adjustment brought about by variation of national income, this adjustment would not be *necessarily* instantaneous. A momentary difference between savings and investment would appear with any lag. So traditional analysis must be overthrown; for, whatever happens, equality $I = S$ *must* be kept constant in time. Savings and investment are identical and not simply equal to each other. Therefore functions

$$S = S(Y)$$

and

$$I = I(Y)$$

are themselves identical. The same curve represents both.

In short:

(i) It is correct to say that national income must be determined *via* equilibrium analysis.

(ii) In this context, equilibrium relates to the quantities of total supply and total demand.

(iii) Quantity $(Y - D)$ equals $(S - I)$. The criterion for equilibrium, then, is, indifferently, $Y=D=(C+I)$ or $S=I$.

(iv) However, KLEIN disregards identity $Y \equiv D$ or $S \equiv I$. He claims that the identical terms can vary in relation to each other.

$$D > Y, I > S: \text{ increasing income}$$
$$D < Y, S > I: \text{ decreasing income}$$

These inequalities contradict definition $Y \equiv C+I$.

(v) Points (iv) and (iii) seem to collide. How can identical quantities mutually adjust? If they do, must we relinquish either the determination of income or at least its determination by equilibrium analysis?

(vi) As a matter of fact, we can hold on to everything. National income is actually determined by balancing total supply and total demand. But the adjusted quantities are virtual and not real. For although Y is identical to D by definition, we have

$$Y^* \neq D^*.$$

Expected demand is autonomous with respect to virtual income; Y^* can be freely adjusted to D^*.

(vii) The determination of income, effected as it is by adjustment between virtual Supply and Demand, depends exclusively upon static analysis. The output of each particular period must be equalised to the expected demand which applies to this output.

(viii) The value of national income for any period p_n is determined by $D^*{}_{0_{pn}}$, the expected Demand for pn's output.

$$Y_{pn} = Y^*{}_{0_{pn}} = D^*{}_{0_{pn}}$$

(ix) Let $Y^*{}_0$ represent the equilibrium of virtual income and Y_0 the equilibrium of realized income. What again is the meaning of these two equilibriums?

—Equilibrium of virtual income is reached by equalizing Y^* and $D^*{}_0$.

—Expression Y_0 is clearly meaningless. It is quite illusory to look for Y_0, the 'equilibrium value' (?) of income in a series where all incomes are realized.

(x) We cannot overemphasize the fact that the idea of dynamic equilibrium cannot be taken over from price theory to the theory of national income determination.

At the beginning of our criticism, we pointed out that KLEIN was well aware of virtual quantities. The passage already quoted makes this quite clear. "The observed levels of savings and investment are those two values on the schedules corresponding to the observed level of income. All the other values of savings and investment along the schedules are not observed; they are virtual levels of savings and investment corresponding to levels of national income other than that level which actually takes place."

But KLEIN was led to confuse quantities S^*, I^*, Y^* and C^* with S, I, Y and C which stand for realized variables. In his subsequent reasoning, he then applied to real quantities laws which govern only virtual quantities. Had he felt compelled to write S^*, I^*, Y^* and C^*, he would have known that the determining function of national income is $D^* = D^*(Y^*)$ and not $D = D(Y)$, since $D = D(Y)$ reduces to $D = Y$. And, as a corollary, he would have recognized that KEYNES' theory is not static simply because of an analytical imperfection: it is necessarily static, by its nature, since Y is determined by virtual quantities.

GARDNER ACKLEY

The consumption function derives its theoretical importance from its use in equation $Y = C + I$. If

$$Y = C(Y) + I(Y)$$

were not considered as a "seminal" equation (SAMUELSON), crucially important for the determination of national income, function $C = C(Y)$ would lose much of its impact.

Actually, we must choose between two alternatives.

1° Function $C = C(Y)$ participates in determining national income, and thus makes a great contribution to the *General Theory*.

2° Function $C = C(Y)$ steps in *after* national income is already determined. All it does is to divide a given income into consumption and investment. In this case, the function is of secondary importance, at least in macroeconomics. Even if no functional relationship obtained between variables C and Y, national income would still be perfectly determined.

ACKLEY finds function $C = C(Y)$ so important only because he considers this function as a co-determinant in national income (I). However, we know that function $C = C(Y)$ cannot logically participate in determining national income. Consequently, the theoretical model of national

income determination as presented by ACKLEY is not only incomplete since no consumption function is statistically ascertainable, but must be totally relinquished (II).

I. "The three preceding chapters have considered the basic Keynesian thesis that consumption is a stable function of income. Our conclusion was a tentative and qualified acceptance of the hypothesis. We can admit the possibility of occasional unpredictable and major shifts in the consumption function, its probable inaccuracy as the basis for sharp short-run forecasting, and the undoubted relevance of other variables (without being entirely sure which these may be, nor how important their net effect). Yet we might still agree that a relationship exists which is of major importance for the understanding and prediction of movements of national income and output. Because the relationship is clearly not precise and unchanging, the theoretical model presented at the beginning of Chapter X, based on a consumption function, will offer, at best a simplified approximation of the behavior of the economy. Nevertheless such simplifications are often useful; indeed, it is the whole purpose of economic theory to simplify, to sort out the major from the minor influences."[26]

This text stresses the importance of the consumption function in the context of macroeconomic income determination. Several authors before KEYNES, for instance Alfred MARSHALL and J. M. CLARK (see p. 218 of ACKLEY's book), had noticed how total consumption relates to national income. But these authors are not real precursors of post-Keynesian thought, for they considered national income to be given in the short-run. "But, in the short run, *aggregate* real income was simply not a variable with which they needed to be concerned." (p. 218) Now, if total income is an exogenous variable, the consumption function intervenes when income is already determined, which lessens considerably the scope of this function. Function $C=C(Y)$ thus gives data, not about the determination, but about the division of income.

Like the other members of the dominant school, ACKLEY thinks that national income is responsive to variations in total consumption.

—*Post-Keynesian hypothesis.* Variation in C brings about variation in Y.

—*Classical or neoclassical hypothesis.* Variation in C brings about an adverse variation in I, so that national income remains unchanged.

If ACKLEY had held to the second hypothesis, he would not have tried with such scrupulous care to provide an experimental basis for the existence of the consumption function. This function seems indispensable

to him because he holds to the assumption made by the dominant theory, that short-term fluctuations of national income cannot be predicted without knowledge, at least approximate, of the total consumption value as determined by function $C=C(Y)$. If, as the author admits, the form of this function is indefinite and unstable, then it should be used as a makeshift until science provides us with a more reliable consumption function. If the function were discarded altogether, short-term variations of national income would remain utterly indeterminable. ACKLEY posits the existence of a consumption function, although it might be only "a simplified approximation of the behavior of the economy."

II. If we assume the existence of function $C=C(Y)$, can we agree with ACKLEY in his explanation of the simplified Keynesian model?

"Algebraically,

(1a) $$c=.8y$$

(2a) $$y=c+i$$

where c is real consumption expenditure, again in the sense of desired or intended consumption. Substituting from (1a) into (2a) gives:

$$y=.8y+i$$

$$.2y=i$$

$$y=5i.$$ "(p. 211)

We could immediately object if the author had not included in his text the difference between desired or intended consumption and realized consumption.

(i) Since $y=c+i$ is an identity, equation (1a), $c=0.8y$, implies $i=0.2y$.

Or, consider identity between savings and investment. ACKLEY is well aware that function $c=0.8y$ either implies function $s=0.2y$ or is implied by it. (See p. 210) Now, equation $y=c+i$ is an identity for the same reason as equation $y=c+s$. The result of $c=0.8y$ is then identically $s=0.2y$ and $i=0.2y$.

By combining the two equations (1a) and (2a), we obtain simply the definition of national income, without having made any analytical headway: $y=0.8y+0.2y$.

(ii) ACKLEY claims that his analysis avoids criticism. His equation $y=c+i$ is not an identity but an equilibrium condition since c and i

are desired and not realized quantities. "This [distinction] makes equation (2a) an equilibrium condition rather than a definition of income. The sum of actual, realized consumption and investment is identical with income. The sum of the desired or intended expenditures does not equal income by definition, but only in equilibrium." (Footnote, p. 211)

The argument is supported by the following chart.

	Original equilibrium level (Before injection)	First round: injection	Second round	Third round	Fourth round	Eventually approaching (New equilibrium)
c	80	84	87	89.25	90.9375	96
y	70	70	73	75.25	76.9375	82
i	10	14	14	14	14	14
$s\,(=y-c)$	10	14	14	14	14	14
$\bar{s}\,(=-10+.25_{y_t})$	10	11	11.75	12.3125	12.734375	14
$s_R\,(=y_{t-1}-c_t)$	10	10	11	11.75	12.3125	14

(p. 321)

The letter $\bar{s}$ does not mean that real saving is a constant. The bar distinguishes desired saving, $\bar{s}$, from realized savings, s. ACKLEY has every right to write $\bar{s}=-10+.25_{y_t}$. While the whole chart seems unobjectionable, its *interpretation* is liable to misunderstanding.

The result must be that either saving $\bar{s}$ is equal to savings s and the desire to save is exactly realized by buyers, or a gap opens up between the desire to save $\bar{s}$ and realized savings s.

In the first case, $\bar{s}=s$, the behavior of the saver influences demand, for it defines the sum of consumer goods sold by firms. When desires are *realized*, their results can be observed. Yet, for $\bar{s}=s$, we have $\bar{s}=i$. Therefore, realized investments can be neither less nor more than the savings desired by buyers. The desired behavior of the saver can have no effect on national income when $\bar{s}$ equals s.

In the second case, where $\bar{s}\neq s$, savings $\bar{s}$ (as such, a desired behavior) has no positive consequence, since it is not realized. On the other hand, saving s is realized. Desired quantities and realized quantities do not coincide. The problem, however, is to explain variations in national income. What factor makes real income vary? The factor may be psychological or behavioral. But a factor which is *only* desired and not realized can have no effect, positive or negative. To have an effect the factor must be realized. Firms have no way of knowing unrealized desires of buyers. They can only react to behaviors which they have

checked on different markets. The only person who can observe a pure desire, like $\bar{s}$, is the one who feels it. Firms, of course, cannot adjust production to the unrealized behavior of consumers. In other words, only savings s can influence a firm's production. Savings $\bar{s}$ could exert no influence, since $\bar{s}$ by definition is an unrealized desire, unknown to business.

The two cases, $\bar{s}=s$ and $\bar{s}\neq s$, give the same result. No amount of desired savings can affect the level of national income. Saving $\bar{s}\neq s$ does not produce the desired effect, because it is not realized. And saving $\bar{s}=s$ is realized; for this reason it involves equality $s=i$ which again precludes any adjustment process.

If we refer to the chart p. 90, we see that realized values of savings and investment are everywhere equal. Therefore no adjustment can take place between these two quantities to explain variation of income after the initial injection. But ACKLEY, who insists on an adjustment, adds a line for desired savings, $\bar{s}$. Between desired savings and realized investment, a disparity shows up in each column, up to the final equilibrium of national income. Thus, in the second round, desired savings are $\bar{s}=11.75$, while realized investment is $i=14$. Since investment is greater than savings, national income increases. But here again, national income is subjected to a purely illusory factor of growth. Realized behaviors in the second round, and at all other times too, immediately equalize savings and investment, $s=i=14$. Savings $\bar{s}$ alone trail, for they represent "'desired saving' — the amount of saving which consumers would desire to make at the going level of income, assuming that level of income were to prevail long enough for them to adjust their consumption (and saving) to that level." (p. 321) In the second round, consumption is equal to $87-14=73$ and not to $87-11.75=75.25$. The real behavior of consumers makes total savings equal to 14 and not to 11.75. The difference between $i=14$ and $\bar{s}=11.75$ is, in the end, completely unreal. The gratuitous $\bar{s}$, an assumption arbitrarily manufactured by the author, has no effect on national production. ACKLEY's cause and effect logic is upside down. When income varies, $\bar{s}$ may lag behind realized savings. But the supposed variation in national income cannot be due to unfulfilled desires.

And yet, on ACKLEY's chart, we can see that national income is increasing, from 84, its level at the moment of injection, to 96, its "new equilibrium." Now the author agrees that it is impossible to attribute this increase to the difference between realized quantities of saving and investment, since these variables are identical. ACKLEY seeks the cause of the increase in the difference between investment and *desired* saving.

This is supported by an initial proposition which seems unquestionable: income increases from 84 to 96. But since this proposition is not an axiom, it must be proved. Does income increase? In other words, has ACKLEY really found a law which requires the increase in national income? Or, did he not unjustifiably assume the increase in income, striving to corroborate it afterwards?

ACKLEY plainly engineered the growth of national income from 84 to 96, for no real factor is at work. Factor $i - s$ is zero. Factor $i - \bar{s}$ is positive but fictitious, since by definition it is not realized. The only remaining factor is the author's will.

ACKLEY first injects 4 units of money so that income rises from 80 to 84. From then on, he applies the dominant theory, by which an induction takes place. The induction is governed by a 'marginal propensity' implicit in $\bar{s} = -10 + .25y_t$. If savings absorb a quarter of the increments in income, consumption absorbs the rest. The induction coefficient is then equal to 3/4. In the second round, income goes up to

$$80 + 4 + {}^3/_4 4 = 87.$$

This first induced increase, $87 - 84 = 3$, is a new multiplicand which must be multiplied by coefficient 3/4. The second induced increase raises national income from 87 to 89.25. And so on.

This whole chain of inductions is fictitious. In the first round, when income is increased by 4 units, analysis offers a choice between two contradictory alternatives:

(i) A discrepancy arises between Demand and Income. Therefore income increases throughout the next rounds.

(ii) No disparity occurs, so that income is not subjected to any growth factor.

Case (i). We would have to write

$$c + i \neq y.$$

Thus, for $c + i > y$, we would have a positive growth factor. Now the chart dismisses this inequality, since for the first round it states $c + i = 84$ and $y = 84$.

Case (ii). Demand $c + i$ is always equal to the corresponding income y. Therefore income cannot vary. Or, to be more precise, *no law of variation is given by function* $\bar{s} = -10 + .25y_t$. *If income grows anyway, it does so thanks to Ackley who is trying to corroborate the dominant theory.* Income variation is merely due to a *deus ex machina.*

The series

$$y = 80, 84, 87, 89.25, 90.9375, \ldots, 96$$

is arbitrary, except for the first two terms. The whole chain of induced incomes lacks basic reality.

ACKLEY, a good logician, should have been alarmed by his own conclusions. The last line of his chart refers to savings in Robertsonian analysis. ACKLEY judges correctly that ROBERTSON's analysis is not based on a psychological propensity. "Specifically, the Robertsonian definition involves no 'psychological' magnitudes at all." (p. 322)

If equation $\bar{s} = -10 + .25y_t$ could explain national income variations, this causative factor would be psychological, since $\bar{s}$ is a desired saving. Psychological quantities should then be the determining factor. In other words, since ROBERTSON's analysis includes no psychological factors, his results should be different from ACKLEY's, if it be true that ACKLEY's analysis is really based on psychological factors. But, on the chart, ROBERTSON's analysis gives exactly the same results as ACKLEY's. Savings s_R add up everyplace to just the same figure as savings $\bar{s}$, as is apparent when we shift one of the last two lines by one round.

When ACKLEY criticizes ROBERTSON for having found no law of income variation, since the differences between savings and investment are the result and not the cause of this variation, this same criticism can be aimed at his own theory. Both *assume* that national income varies in time; they do not prove it.

Before leaving ACKLEY, let us note again how he struggles against identity $Y = C + I$ or $y = c + i$. He does not object to identity between total supply and demand. He completely credits this identity. "There are clearly two senses (or more) in which we can speak of an equality between saving and investment. In the sense in which these terms are defined for national income accounting purposes, it is clear that saving and investment must always be identically equal." (p. 320) Savings equal investment; for total supply, y, is necessarily equal to total demand, $c + i$.

However, ACKLEY would like to have it both ways. On the one hand, he approves the identity. But on the other, he considers it important to safeguard equation $y = c + i$ as a *condition of equilibrium*. For economics must have a law of adjustment between total supply, Y or y, and total demand, $C + I$ or $c + i$.

How can we reconcile these two apparently contradictory statements?

$y = c + i$, condition of equilibrium

$y = c + i$, identity

We know that we need only write

$y = c + i$, realized quantities: identity

$y^* = c^* + i^*$, virtual quantities: condition of equilibrium.

Identity can only be broken by analyzing virtual instead of real quantities. Yet ACKLEY tried another method. He considered it sufficient to introduce desired instead of virtual quantities. Now we have already shown that the concept of desired savings leads us into a blind alley. Will the concept of desired investment be more effective? ACKLEY writes

$y=c+i$: identity, and
$y=c+\bar{i}$: condition of equilibrium

where $\bar{i}$ stands for desired investment.

	Original equilibrium level (Before injection)	First round: injection	Second round	Third round	Fourth round	Eventually approaching (New equilibrium)
y	80	84	87	89.25	90.9375	96
c (sales)	70	73	75.25	76.9375	78.203125	82
c (production)	70	70	73	75.25	76.9375	82
$\bar{i}$	10	14	14	14	14	14
$s\ (=\bar{s})$	10	11	11.75	12.3125	12.734375	14
i	10	11	11.75	12.3125	12.734375	14

"In line five, we show saving, which is, it will be noted, the same whether defined in the national income sense or as 'desired' saving. This is so because we assumed away (for this example) any lag between income change and consumption change, that is, any possible difference between desires and their fulfillment. But it is clear, by comparison with the investment figures in row three, that there is divergence between s and $\bar{i}$ except at equilibrium. This is because the investment figures relate not to actual, realized investment (in the national income sense) but to planned or intended investment. (This is why we have introduced the bar above the symbol.) Actual investment is shown in row six. It includes the change in inventories (as we do in national income accounting) whether this change was an intended, deliberate change, or whether —as in this case—the inventory change is an accidental result of a rise in demand which producers did not expect and had therefore not prepared for. Once again, there can be no escaping the equality of s and i at each and every point in the process, equilibrium or no equilibrium. But it is also not meaningless to talk about a *difference* between saving and investment (defining investment now in a different way) and their equality as a condition of equilibrium." (p. 323)

In attempting to change identity $y=c+i$ into a condition of equilibrium, ACKLEY not only holds to it but he adds a *second* identity, $c+\bar{i}=y$.

The chart below shows *two* identities $y=c+i$ and $y=c+\bar{\imath}$.

Real incomes	80	84	87	89.25	90.9375	96
$c+i=y$	70	73	75.25	76.9375	78.203125	82
	+10	+11	+11.75	+12.3125	+12.734375	+14
	80	84	87	89.25	90.9375	96
$c+\bar{\imath}=y$	70	70	73	75.25	76.9375	82
	+10	+14	+14	+14	+14	+14
	80	84	87	89.25	90.9375	96

The two equations $y=c+i$ and $y=c+\bar{\imath}$ are in force for *any value of* y and not only for equilibrium values $y=80$ and $y=96$.

ACKLEY's two consumptions, consumption (sales) and consumption (production), far from ridding us of an unwanted identity, land us instead with two.

$$c \text{ (sales)} + i \equiv c \text{ (production)} + \bar{\imath} \equiv y$$

The distinction between realized investment i and desired investment $\bar{\imath}$ degenerates into the distinction between the two consumptions:

$$i \equiv y - c \text{ (sales)}$$
$$\bar{\imath} \equiv y - c \text{ (production)}$$

1° It is correct to distinguish between the two consumptions, c (production) and c (sales). We can write c' for the real output of consumption goods and c for the final sale of these goods.

2° In the same way i (production) and i (sales) can be differentiated. ACKLEY does not do this in his chart. When he opposes realized investment to desired investment, he winds up again with the simple distinction between the two *consumptions*, c and c':

$$i - i' = c \text{ (production)} - c \text{ (sales)}.$$

3° Now the two consumptions c and c' are realized quantities. Therefore they must be equal to each other.

$$c \equiv c'$$

More generally,

$$c+i \equiv c'+i'.$$

The above equivalence results from $y \equiv c+i$, for y is the supply of real output, subdivided into supply of consumption goods c' and supply of investment goods i'.

Note that ACKLEY is fully aware of the two identities $c \equiv c'$ and $i \equiv i'$. In his chart, i stands for both i and i'. On the other hand, c is always equal to c'. To show this, all we have to do is shift consumption (production) one round relative to consumption (sales).

4° Lastly, we can demonstrate that ACKLEY never at any time writes equation $y=c+i$ as a condition of equilibrium. The substitution of desired investment for realized investment has no effect, for equation $y=c+\bar{i}$ is not a condition of equilibrium either.

Take the income of any period, of the second round, for instance. It is equal to 87.

How far does consumption account for this income? The answer seems to depend on our definition of consumption. Must we reckon with consumption (production) or consumption (sales)?

ACKLEY writes income with a small letter, since he intends to treat real and not nominal income. Therefore we can say straight off that the relevant variable is consumption (production).

Accordingly, in the second round, consumption accounts for 73 out of the 87 income units.

However, instead of consumption (production), we can still use consumption (sales). Given identity $c' \equiv c$, the result must remain unchanged. Nevertheless we have to watch our step. Only sales of consumption goods produced in this same period must be counted, even if these occur in another period. Since consumption (sales) precede output by one period, second-round consumption (sales) equal 73 units.

Whether we use consumption (production) or consumption (sales), the result is the following:

consumption (second period) = 73 units.

ACKLEY's mistake shows up immediately on the line of realized investment. In the second round, realized investment equals income (87) minus consumption (73), that is, 14 and not 11.75. In all rounds except the last, figures for realized investment must be corrected. They must always be equal to the value of desired investment.

5. Increase of income as presented in the first line of the chart is completely unjustified. It cannot be derived from any known law.

No force of adjustment can be found in the supposed disparity between realized and desired investment. By ACKLEY's own analysis,

logically worked out, no difference can arise between these quantities.

How would a gap open up between consumption (production) c' and consumption (sales) c? Identity of c' and c inevitably results from identity $y \equiv c+i$, or identity $c'+i' \equiv c+i$.

Again, the time trend of income toward its 'equilibrium value' is pure fiction, for any value of y in series

$$84, 87, 89.25, 90.9375, ..., 96$$

is as 'balanced' as any other.

ACKLEY's analysis is, then, kinematic and not dynamic. With no valid explanation, he assumes successive incomes in increasing order. He could just as well have assumed a constant or a decreasing income. Even if income varies erratically, identities

$$c' \equiv c$$
$$i' \equiv i$$
$$y \equiv c+i$$

hold firm under the wildest assumptions.

*
* *

Before concluding our criticism of ACKLEY, we shall review some elementary propositions.

1. Equation $Y=C+I$ or $y=c+i$ is an identity. It is solved for any value of national income, irrespective of the distinction between desired and realized quantities.

2. Determination of the so-called dynamic equilibrium value of national income is obviously unthinkable under these conditions. For what single value of income is Y equal to $C+I$? This question is purely nonsensical. Any value of Y is compatible with equality between Y and $C+I$.

Equality $y=c+i$ as a condition of equilibrium must be the product of an entirely new definition of total supply and total demand.

3. *Like all authors of the dominant school, Ackley bases income analysis on the supposed price theory analogy.* (See p. 320)

On the strength of this analogy, why does ACKLEY have to distinguish between desired quantities, $\bar{s}$ and $\bar{\imath}$, and realized quantities, s and i?

In price theory, distinction between realized and desired supply and demand has no meaning. All offers and all demands, even in response

to purely virtual prices, are real quantities. If the consumer wants to buy x units of a commodity at price level y, demand xy is not simply desired, it is real. May be the potential transaction will never take place. If not, is demand xy real, or only desired? In other words, does it define observable behavior, or is it only an ineffective psychological propensity? Let us try to clarify the issue by examining separately the consumer's and the seller's behaviors. Behavior measured by xy is real and observable. The consumer really sets out to buy x units of the commodity at price level y. Demand and supply are always real behaviors, although their convergence remains purely virtual up to the point of equilibrium.

However, in his Chapter V, "Say's Law and the Quantity Theory of Money," ACKLEY attempts to introduce into price theory his distinction between realized supply and demand and desired supply and demand. The following is the kernel of his argument.

"In it [Figure 5.1] we show supply and demand functions, intersecting at a point which 'determines' price and quantity.

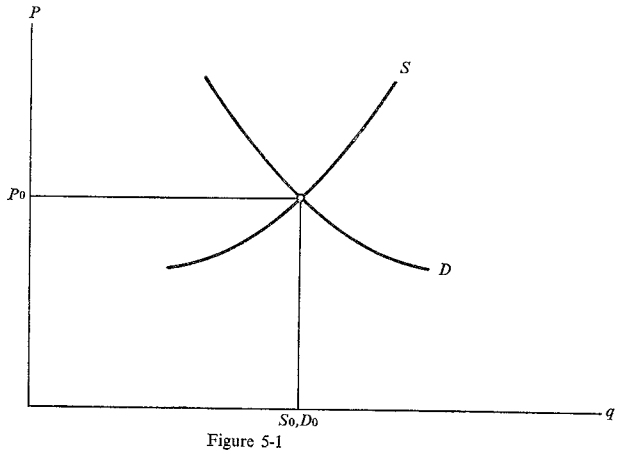


Figure 5-1

Fig. 26

"Now it should be clear that the relationship we have here described between amount bought and amount sold is a different one from the definitional identity. Here we have amount sold as a function of price:

$$\overline{S} = S(P)$$

amount bought as another function of price

$$\overline{D} = D(P)$$

and an equilibrium condition

$$\overline{S} = \overline{D}$$

which corresponds to the intersection in the diagram. Note that we did not write

$$\overline{S} \equiv \overline{D}$$

Clearly, we have introduced some subtle difference of definition which makes S and D here mean something other than what they meant in the formulation that finds them identical. This is why we did not use the same symbols. Rather, $\overline{S}$ represents the quantity that sellers wish to supply, and will sell if they get a chance. $\overline{D}$ represents the amount that buyers desire to purchase, and will purchase if they can. As functions of price, these amounts can clearly be different, and are different, at most (or even conceivably at all) prices, although amounts *actually* bought and sold must be identical. To write as a 'condition of equilibrium' that $\overline{S}$ must equal $\overline{D}$ is to say that if the price were to be at any level other than the P_0 in the diagram, either buyers or sellers (or both) must be disappointed—must be buying or selling more or less than they would wish to buy or sell at that price. This disappointment would lead them to do something different than they are in fact doing: that is, any price other than P_0 would not represent an equilibrium situation. Since S necessarily always equals D, if $\overline{S}$ were unequal to $\overline{D}$, either $\overline{S}$ would be unequal to S or $\overline{D}$ unequal to D. Equilibrium requires that both buyers and sellers be satisfied with their behavior, i.e., that

$$\overline{S} = S \text{ and } \overline{D} = D$$

or, since

$$S \equiv D$$

equilibrium requires that

$$\overline{S} = \overline{D}$$

If (as is usual) we assume that a stable equilibrium exists, and if our concern is only with equilibrium, we neglect the distinction between $\overline{S}$ and S, $\overline{D}$ and D, and we never are confused as to whether we should say that supply equals demand or is merely identical with demand. (Strangely, however, in a strictly analogous piece of analysis, some economists have argued endlessly whether saving equals investment or is merely identical with investment, apparently believing that if it is one relationship it cannot also be the other!)" (pp. 106–108)

1° The vertical axis measures the virtual price level. Since ACKLEY is not aware that we are here concerned with unit prices, he proposes to

derive two determinations from supply and demand curves, price and quantity. Actually, adjustment between supply and demand is a single factor, capable of determining only one quantity. Only one variable is determined, namely price itself, which is the product of the equilibrium unit price and the physical quantity of the exchanged commodity.

2° Our principal criticism bears upon the distinction between S and $\overline{S}$ and between D and $\overline{D}$, which ACKLEY relies on.

—When he so distinguishes, his intention is not to facilitate price analysis but to lay the foundation for national income analysis. His aim is to handle equation $Y=C+I$ or $S=I$ as a condition of equilibrium, although the two terms are identical to each other in national accountancy.

—We have to move from identity $Y=C+I$ or from $S=I$ to a *causal* relationship. In economic analysis, tautological identities are not effective. How are we to conceive of inequality $I>S$, for example? To reach beyond tautology we must first solve this problem. If investment can be greater than savings, economic analysis will dispose of an instrument far more powerful than accounting identities, capable of explaining the dynamic growth of income. "The equality which we discovered in Chapter II among total spending, the money value of total output, and (under simplified assumptions) total income was only a definitional identity. It implied nothing whatever as to causation. Its existence does not mean that we can conclude that output *determines* spending, or that spending *determines* output. In effect, all that this identity reflects is the equality between amounts purchased and sold. The equality between saving and investment is of precisely the same character, although one stage removed from the identity of purchases and sales." (p. 105)

If we are looking for an *adjustment factor* between total supply and total demand, or between S and I, then our first move must be to split the identity of Y and $C+I$. ACKLEY's distinction between desired and realized quantities does not answer the purpose, as we have already shown, for it does not succeed in upsetting the identity of Supply and Demand.

3° *a.* There is no reason for differentiating between desired and realized supply. *b.* National income is not the sum of microeconomic prices. *c.* Determination of national income requires virtual quantities.

a. Analysis of price determination does not involve distinction between desired and realized quantities.

The two functions represented in Figure 26 are

$$D=D(P)$$
$$S=S(P)$$

and not

$$\overline{D}=\overline{D}(P),$$

$$\overline{S}=\overline{S}(P).$$

It is in no way contradictory to write

$$S \equiv D \text{ (for realized purchases)},$$

$$S=D \text{ (condition of equilibrium)}.$$

As a matter of fact, equality as a condition of equilibrium refers to phase *ex ante*. In this phase, supply and demand become equal only for a certain value of price, when $P=P_0$.

ACKLEY's preoccupation is thus unfounded. He writes

$$\overline{S}=\overline{D} \text{ (condition of equilibrium)}$$

as opposed to

$$S \equiv D \text{ (identity)}.$$

The same quantities, S and D in ACKLEY's explication, form an identity or a condition of equilibrium according to the instant in real time at which they are considered. If we choose the instant of purchase, we have an identity and not a condition of equilibrium. On the contrary, if we choose a moment either before or after purchase has taken place, we have a condition of equilibrium and not an identity.

Of the following arguments, any one would suffice to show how useless it is to distinguish between S and $\overline{S}$ and between D and $\overline{D}$.

—The distinction would have meaning if the desired quantities were not simultaneously real quantities. If the consumer wants to buy quantity x of the commodity at unit price y, xy obviously measures his desire to buy. But it likewise measures real demand. ACKLEY cannot help granting this. If the seller accepted price xy, the transaction would be concluded. The buyer actually wants x at price y even if nothing comes of it. "$\overline{D}$ represents the amount that buyers desire to purchase, and will purchase if they can." This quantity $\overline{D}$ is none other than quantity D, the demand really expressed by the buyer when price is at level $P=y$. In the same way, we have identity $\overline{S} \equiv S$.

—The buyer not only wants to demand, he really does demand. Similarly, the seller does not simply wish to supply. He means business. What supplier and buyer really desire is not merely to offer and to demand, for they do just that at even purely virtual prices, but they hope that their behavior will lead to a realized transaction. *Ex ante*, supply and demand are autonomous factors. *Ex post*, supply and demand form a single

factor. In short, distinction between desired and real quantities applies to 'supply-demand' and not to the separate factors of supply and demand.

—ACKLEY writes

$$\overline{S}=\overline{D}$$

in order to avoid identity

$$S \equiv D.$$

However with every realized transaction, the desires of buyer and seller coincide, so that we must write

$$\text{Realized transaction} \begin{cases} S \equiv D \\ \overline{S} \equiv \overline{D}. \end{cases}$$

This in itself gives sufficient proof of identities $S \equiv \overline{S}$ and $D \equiv \overline{D}$. The identity of supply and demand in all realized transactions connects $\overline{S}$ and $\overline{D}$ in the same way as S and D. A switch from quantities S and D to quantities $\overline{S}$ and $\overline{D}$ is neither required nor has it any effect. The very same quantities refer to a condition of equilibrium or to an identity, according to the chosen time point. *Ex post*, we must write identity $S \equiv D$. *Ex ante*, we have $S \not\equiv D$, for then the two terms are equal only at equilibrium.

b. National income is not the sum of microeconomic prices.

"Despite the fact that these accounting relationships imply nothing as to causation, it is of course possible to postulate causal relationships, direct or indirect, among the same variables. This moves us from the realm of accounting into economics. The difference can probably be seen more clearly if we consider at first not broad aggregates like total output and total spending, but instead consider some analogous micro-economic concepts, such as sales and purchases of a single commodity." (p. 105)

The analogy between the macroeconomic determination of income, that is the adjustment between total supply and total demand, and the microeconomic determination of prices, the adjustment of supply and demand, implies that national income is nothing but the aggregation of individual prices.

But national income does not belong to the price family.

—Expression $Y \equiv C+I$ can just as well be written $C'+I' \equiv C+I$. ACKLEY himself distinguishes between C (production) and C (sales). Supply Y, the production of goods, is equal to $C'+I'$, the total remuneration of the factors of production. So, $Y=$ value of production $=C'+I'$.

But microeconomic sales do not necessarily occur at the cost price of goods. If they did, profits could only be zero.

Therefore microeconomic prices include transfers or "profits upon alienation."[27] The sum of prices realized during one period exceeds the corresponding income.

—By far the greatest theoretical divergence between microeconomic prices and national income is the following: exchanges are without a time dimension while income can only be produced in a finite period. The conclusion of any transaction is instantaneous, so that the sum of realized prices (purchases) occupies only a finite number of instants. On the other hand, the production of national income extends over the entire period. Thus it takes a whole day to produce a day's income. The production of national income for any finite period spreads over all the instants included in that period, that is, over an infinite number of instants.

This difference between the time dimension of the two phenomena leads to crucial theoretical results.

c. Determination of national income depends on virtual quantities.

Proof: Supply and demand of any commodity are autonomous with respect to each other at any moment except when purchases are realized.

Supply and Demand in national income analysis are never autonomous with respect to each other. They are identical at each instant of real time.

In order to keep to the analytical method of reasoning by supply and demand, we have to proceed from realized quantities to virtual quantities. Equilibrium income solves

$$Y^* = C^*_0 + I^*_0.$$

Adjustment of Y^* to $C^*_0 + I^*_0$ is logically possible, for the necessary equality of total supply and total demand does not apply to virtual quantities. Value Y^*_0 is obtained by adjusting virtual national income to the sum of expected demands.

Implications: We shall discuss here only the consumption function. In a given economy, is it possible to predict the exact amount of total consumption from the knowledge of national income? The answer is certainly negative if we look for a function in the exact mathematical sense. We could always, of course, 'manipulate' the definition of income so that it would fit our preconceived consumption function. For this purpose, we could replace current income by some other concept, like permanent income. But even by tampering, by slanting an experiment toward a preconceived result, we could never set up function $C = C(Y)$ in the true mathematical sense. Too many factors interfere, alongside income. Serious doubt may be cast on the consumption function by recourse to an entirely different argument. The dividing line between consumption goods and capital goods can never be strictly drawn.

In spite of these difficulties, let us assume that we can write function $C=C(Y)$. Whatever its mathematical form, the *consumption function cannot affect determination of national income.* Sequence I below is logically tenable but Sequence II is not.

Sequence I. The equilibrium of virtual income is determined by equalization of virtual Supply and virtual Demand. Value Y^*_0 measures the total amount of national income which will actually be produced, Y. Once it is determined and only then, income is divided into consumption and investment. Expenditures $C+I$ relate only to the income from which they proceed.

Sequence II. Let us give a numerical example. Expenditures $C'_1+I'_1$ are wholly autonomous: $C'_1=50$, $I'_1=I_0=30$. Income Y_1 is thus equal to 80. Right here the consumption function intervenes. Suppose it is linear, with $C_0=50$.

$$C=C_0+{}^1/_2Y$$

In this case,

$$C_2+I_2=90+30=120.$$

The new income, $Y_2=120$, induces a consumption equal to 110. Thus

$$C_3+I_3=110+30=140.$$

Income increases up to equilibrium value Y_0.

$$Y_0=(C_0+I_0)\cdot\frac{1}{1-{}^1/_2}=160$$

The importance of the consumption function is evident in this example, for it enters into national income determination.

Sequence II is erroneous. The consumption function in no way contributes to the determination of national income. True, expenditures C_1+I_1 are equal to the corresponding income Y_1. But note first that income does not result from these expenditures. On the contrary, expenditures result from income. Expenditures C_1+I_1 do not produce income Y_1. They simply make up the other side of the already given income. Thus no connection between income Y_1 and expenditures C_2+I_2 can be established. These new expenditures correspond to the new income Y_2, but not to the preceding income Y_1. Once corrected, Sequence II dissolves into a series of independent links. The chain is entirely broken into bits and pieces.

$$[C_1+I_1 \rightleftarrows Y_1] \quad [C_2+I_2 \rightleftarrows Y_2]\ldots$$

Without a single exception, expenditures refer back to the income out

of which they sprang. They neither produce new income nor do they in any way relate to it.

Expenditures C_1+I_1 have no connection with income Y_2, for they are expenditures out of income Y_1 and not out of Y_2. No bridge stretches between income Y_1 and expenditures C_2+I_2. If there were such a bridge, it would span the distance between expenditures C_1+I_1 and income Y_2. *Now no such path exists between income Y_1 and expenditures C_2+I_2.* Perhaps this might be questioned on the strength of our consumption function, $C=50+{}^1/_2Y$. But income Y_1 equals 80. The sum of the corresponding consumption is $50+{}^1/_280=90$. If these figures are correct, we must have

$$\left.\begin{aligned} Y_1&=80\\ C_1&=90\\ I_1&=30 \end{aligned}\right\} Y_2.$$

But the figures cannot possibly be correct. If we agree to $Y_1=80$, and to $C=50+{}^1/_280$, it necessarily follows that investment I_1 is equal to -10. Thus, if $C=50+{}^1/_2Y$, I must be equal to $-50+{}^1/_2Y$, for $C+I$ equals Y for any value of Y. When $Y=80$, $I=-50+40=-10$. Logically then,

$$\begin{aligned} Y_1&=80\\ C_1&=90\\ I_1&=-10. \end{aligned}$$

Quantities C_1 and I_1 are simply the expenditures of income Y_1. They have no connection with income Y_2.

If we look again at Sequence I, we see that national income, Y_1 for period p_1, Y_2 for period p_2, is determined by virtual quantities. Once it is determined, and only then, consumption comes along and so does the hypothetical consumption function. It appears that the consumption function is not nearly so important as the dominant School would have it. Its only achievement is to partition a given sum of income into two categories of expenditure. Pre-Keynesian analysis was correct in holding that national income is not affected by consumption. Any change in consumption is compensated by the adverse variation of investment. The Keynesian theory is, however, original in another respect. National income cannot be taken as given in the short-run. It is determined by $C^*_0+I^*_0$, the sum of expected demands. Determination of national income by virtual quantities sets up an entirely new macroeconomic science.

AXEL LEIJONHUFVUD

"Chapter VI sums up the main line of our argument: KEYNES, in the interpretation offered here, departed from the postulates of Classical doctrine on only one point. Furthermore, the postulate which he relinquished should have been recognized as objectionable in the first place. His model is characterized by the absence of a 'Walrasian auctioneer' assumed to furnish, without charge and without delay, all the information needed to obtain the perfect coordination of the activities (both spot and future) of all traders. That his theory does without the contrived assumption of 'recontracting' means that his claim to having attempted a more 'general theory' is justified."[28]

According to the above passage, KEYNES added nothing essential to economic theory. Macroeconomic analysis subsequent to the *General Theory* was only a generalized model of Walrasian equilibrium. And LEIJONHUFVUD insists that this generalization remedies a deficiency which should never have existed, so obvious was it right from the beginning. Before the "Keynesian revolution," official operators in different markets were assumed to receive immediately and without cost all necessary information, especially about price level and quantities purchased, for coordinating activities and assigning to them their just value with respect to general equilibrium. KEYNES was supposed to have corrected this assumption and to have introduced into analysis an important factor which economists had apparently overlooked: the fact that information, requiring time and cost for travel, introduces lags into the economic adjustment process. All macroeconomic disequilibrium appearing at a certain moment is made good by degrees, so that the economy reaches its new level of equilibrium only after a positive lag.

If KEYNES' contribution were really so small, why did the *General Theory* create such a stir?

We shall examine the core of Alex LEIJONHUFVUD's excellent work. I. The author accepts the teaching of the dominant theory. II. Only if he had noticed the logical error inherent in this theory would his interpretation have shed new light on Keynesian thought.

I. Axel LEIJONHUFVUD does not reject the dominant theory.

"As time goes on, either of two things may happen: (1) Ongoing research leads to the accumulation of anomalous findings that make it increasingly difficult to maintain the orthodox Vision—to use SCHUMPETER's term—of how the world functions.[29] The sharper the cutting

edge of the empirical tools available in the field, the more likely this is to happen. (2) As the empty boxes are filled, new generations of researchers are inculcated with an orthodox Vision elaborated with more and more concrete details. The increasing amount of empirical findings accumulated and organized within the given conceptual framework requires an increased effort on the part of the individual who would learn the 'state' of the field. It also creates more questions than it settles and tends, therefore, to lead to increasing specialization.[30] When this happens, it becomes more and more difficult for the individual to avoid scientific myopia, and to keep his subject in perspective and maintain a dispassionate overview of the entire field. In particular, it becomes difficult to keep in mind that alternative, latent Visions, capable of organizing the same collections of 'facts,' must always exist—or to imagine what these alternatives would be. To appraise the state and prospects of the field comes to require a 'struggle of escape.'

"Surely this second type of process resembles the development of the majority school in macroeconomics more than does the first. It is certainly a situation of this type that SOLOW's quoted statement describes." (pp. 5–6)

In the above quotation, SOLOW states that macroeconomic theory henceforth rests firmly on basic principles. The remaining task of economists, he continues, will only be to fill "empty boxes," that is, to furnish theoretical and formal structures with a growing mass of more detailed empirical information.

LEIJONHUFVUD does not dispute this judgment. He thinks too that the propositions taught by the dominant theory, or the majority school, accommodate perfect internal logic. However, less optimistic than SOLOW, he foresees two possible developments.—The theory will not be able to stand up against new facts revealed by future experimentation.—Details will finally accumulate to such an extent that no synthesis will be possible. The general "Vision" will be lost.

LEIJONHUFVUD never catches on to the mistake in logic which undermines the dominant theory.

II. LEIJONHUFVUD errs in the same way as the dominant theory, for the Keynesian income analysis cannot be dynamized.

LEIJONHUFVUD accepts the theory of the time sequence of incomes, and finds the Keynesian analysis basically dynamic (1°). LEIJONHUFVUD does not grasp the originality of KEYNES' new method of macroeconomic

analysis: determination of national income rests on the virtual factor of effective demand (2°).

1° "'*Pseudo-dynamics*': *Keynes versus Marshall* KEYNES dealt with dynamic processes by means of a 'comparative statics' period-analysis. His employment of a static apparatus has frequently been criticized, but the critique has often suffered from a confusion of the method and substance of the *General Theory:* the subject of his work is not 'unemployment equilibrium' but the nature of the macroeconomic process of *adjustment* to a disequilibrating disturbance. The method attempts to analyze this continuous process with the tools of static equilibrium theory. The device which makes such a method possible involves the conceptual partitioning of the continuous adjustment process into discrete stages or 'periods.' This device was not KEYNES' invention. MARSHALL had made much use of it, and in this aspect of his method, as in many others, KEYNES was very Marshallian. But KEYNES differed substantively from MARSHALL as well as from other price theorists in the use he made of this device." (pp. 50–51)

In LEIJONHUFVUD we can re-examine elementary price theory. Analysis of supply and demand by reciprocal adjustment leads to the determination of pq, where p is price level and q the quantity purchased. But pq is the expression of a single reality, the transaction itself. Note the synonymy between 'purchase' and 'price.'

$$pq = \begin{cases} \text{purchase} \\ \text{price} \end{cases}$$

The important thing is to distinguish between p and q, for pq is the measure of realized price (or value of purchase), while p is only the level of this price.

Knowing this, LEIJONHUFVUD thinks that KEYNES has simply shifted the accent of macroeconomic theory from p to q. "*In the Keynesian macrosystem the Marshallian ranking of price- and quantity-adjustment speeds is reversed:* in the shortest period flow quantities are freely variable, but one or more prices are given, and the admissible range of variation for the rest of the prices is thereby limited." (p. 52)

If LEIJONHUFVUD were right, only a faint line would divide pre-Keynesian from Keynesian thought: pre-Keynesians studied factor p and Keynesians, factor q, of the transaction pq. Is that really all it amounts to? Price analysis requires the study of pq and not of p to the exclusion of q or *vice versa* depending on the theorist's personal preference. KEYNES' real originality consists in much more than preferring factor q to factor p rather than the reverse, like Marshallians.

The *General Theory* does not study prices, but national income determination. Now, no analytical bridge exists which permits us to pass from pq to the definition of national income.

What is the basic distinction between pq and Y? Expression pq stands for purchases. Thus, by itself the pq theory explains an *exchange* economy. To study p and q separately is not an important methodological restriction since any purchase is a composite reality, pq. The personal preference of each theorist is likewise of secondary importance. If some emphasize the study of unit prices p, and others the variation of quantities purchased, q, they all nevertheless study real exchanges, or prices, pq. Any theory devoted to Y has to do with a *production* economy. KEYNES did not simply complete the Marshallian theory. He upset it altogether. He raised analysis to the level of a production or monetary economy at a time when science, even macroeconomics, still remained on the level of a real or exchange economy.

Even in a production economy, price study is certainly worthy of analysis. No theory can be criticized for attempting to explain price determination on different markets. The mistake met with today is of another kind. Many theorists, including all neoclassicists and Marshallians, treat national income only as a generalization of price theory. The second-hand market aside, sale-proceeds would define income for the total population. The fundamental mistake here is uncovered by the Keynesian analysis. National income identifies Supply, $C'+I'$, and Demand, $C+I$. Total demand in national income analysis cannot be defined as the sum of individual demands such as they appear in realized transactions pq. Total demand $C+I$ is smaller than Σpq. It includes only *cost* prices $C'+I'$ of the total goods and services sold.

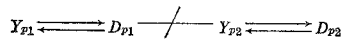
The dynamic or static character of Keynesian theory can be judged by the distinction to be made between Σpq and Y. To carry out all transactions pq during a given period, a finite number of instants is needed. On the other hand, the production of national output extends over the whole period. From this we may infer:

—If the period can be divided into a series of discrete intervals, prices on successive days form a chain in time, since prices realized today can be explained by prices evolved on previous markets. A concatenation is possible because demand can become excessive in any lapse of time separating transactions.

—Division of the period into discrete intervals is appropriate to national income just as to price analysis. Actually, in each branch of production, businesses must take into account the expected, foreseeable demand for their output. Virtual demand probably cannot be calculated

long in advance. The horizon varies with each industrial sector. If, for example, predictions can be made for a period of three months ahead, this does not mean that firms must adjust their production and employment level every three months. Decisions can be made in much shorter intervals, even from month to month. Assume that January forecasts carry over into April. This does not prevent firms from adjusting the number of their employees every month according to expected demand. If the number of workers participating in the total economy be fixed at the beginning of each month, do incomes form a dynamic chain as a result of the principle of expected demand? Surprisingly enough, although prices realized from day to day do form a chain, each income is autonomously produced to fill its expected demand. Thus, income produced in January corresponds to the expected demand for January's output.[31] Income produced in February likewise answers the expected demand for February's output. Incomes would form a dynamic chain only if the demand resulting from income produced in January could induce February's income, or that of another month. Now demand resulting from income produced in January can only correspond to the expected demand for January's output. That is, it has no part in February's income, but only in January's income. As soon as demand is realized, it relates to an income already produced. Realized demand can belong only to the production from which it results. All output must be bought. *Therefore every production finally absorbs all incomes which it has injected.*

The expenditure of realized incomes cannot, then, give rise to new incomes. It can only absorb the old incomes from which it stems. Expectations are, nevertheless, partially founded on the total sum of spontaneous demand currently recorded by each firm. The Keynesian principle of effective demand is in no way affected by this. Whatever sources of information firms may have to implement their forecasts, actual incomes give rise to a total demand just sufficient to absorb them, and are quite incapable of overflowing into supplementary incomes.



Nothing connects successive incomes Y_{p1} and Y_{p2}, since demand D_{p1} is completely exhausted by the purchase of period p_1's output so that it cannot lead to income Y_{p2}. The demand capable of giving rise to Y_{p2} is D^*_{0p2}, the expected demand for period p_2's output.

2° "From this perspective, KEYNES' 'long struggle of escape' seems primarily to have been a struggle with the dynamics of the Marshallian period-analysis. Many critics as well as sympathetic interpreters of KEYNES have had similar difficulties in freeing themselves from the intuitive plausibility of MARSHALL's ranking." (p. 53)

LEIJONHUFVUD does not fully appreciate the "struggle of escape" that KEYNES had to wage against Marshallian dynamics. The revolution is more deep-seated than the mere inversion of ranking between factors p and q in expression pq. The analytical difficulty is to pass from price theory pq to the determination of Y. In the field of national income determination, dynamic analysis is completely out of place. A variable such as Y, since it is determined by the *virtual* factor of expected demand, cannot be analyzed in time. The series of successive incomes Y_{p1}, Y_{p2}, Y_{p3}, etc., cannot undergo dynamic analysis, for each income of the series is fixed by a virtual quantity, expected demands D^*_{0p1}, D^*_{0p2}, D^*_{0p3},... The determination of national income, then, can only be subjected to static analysis. Expected demand for the output of period p_n only determines income for the same period p_n. And analysis referring to a single period must be static.

In the literal sense of the words 'dynamic' and 'static,' national income determination is not dynamic. But neither is it static in the conventional meaning of the word. The introduction of virtual factors into economic theory creates a new methodological category, not to be classed strictly with either dynamics or statics.

"Often in the writings of economists the words 'dynamic' and 'static' are used as nothing more than synonyms for good and bad, realistic and unrealistic, simple and complex. We damn another man's theory by terming it static, and advertise our own by calling it dynamic."[32] In this trivial context, Keynesian analysis would be dynamic, for it is correct and not erroneous, realistic and not abstract, simple instead of uselessly complicated. However, KEYNES has often been criticized for having elaborated a too static analysis. LEIJONHUFVUD correctly opposes this judgment, but his argument seems inadequate: the *General Theory* is static only in its method, in substance it is dynamic; it explains how the economy reacts under the influence of a disequilibrating factor, and then progressively ('dynamically') resumes its equilibrium at a new level of production and employment. But in fact KEYNES' theory proposes a basic adjustment of total supply to total demand which does not take place in real time since virtual income Y^* must adapt to virtual demand D^*_0. The Keynesian analysis is therefore static in substance as well as in method.

The dominant theory falls into a logical error shared by LEIJONHUFVUD: the *General Theory* explains in substance the dynamic setting-up of a new macroeconomic equilibrium after a disturbing shock. Actually, the dominant school, including LEIJONHUFVUD, misses the basic innovation of the *General Theory*. National income is determined by adjustment of virtual quantities, which preclude dynamics.

Dynamic analysis is irrelevant, since realized quantities alone can be followed in real time. "The *actually realized* results of the production and sale of output will only be relevant to employment in so far as they cause a modification of subsequent expectations." (*General Theory*, p. 47) We can assume that income which takes place in t is spent in $t+1$. Expenditure out of Y_t, then, is always made in $t+1$. Suppose that C_{t+1} is the consumption recorded in $t+1$. This consumption may possibly reflect function $C=C(Y)$, so that C_{t+1} is equal to $C(Y_t)$. But whether or not consumption is a function of income, C_{t+1} belongs to income Y_t inasmuch as expenditure C_{t+1} is taken out of income Y_t. Thus, it would be inaccurate to relate consumption C_{t+1} to income Y_{t+1}. Since the consumption corresponding to income Y_{t+1} is C_{t+2} and not C_{t+1}, realized consumption C_{t+1} gives no clue to the new income which firms must produce in either sector during period $t+1$. We know only that consumption C_{t+1}, a fraction of income Y_t, has no connection with income Y_{t+1}. As the quotation from KEYNES points out, realized consumption C_{t+1} can affect the expectation of sales for the new output of period $t+1$. The fact remains that income Y_{t+1} is determined by *one causal factor only, expected demand relative to period $(t+1)$'s output.* In particular, consumption goods produced in $t+1$ will be equal to the expected sales of consumer goods from period $(t+1)$'s output, no matter how firms determine their expectations which may be influenced by actual sale-proceeds on past production. Determination of Y_{t+1} by $(C^*_0+I^*_0)_{t+1}$ occurs entirely in the area of virtual quantities.

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(i) The Keynesian analysis of national income determination is not dynamic, because of the very nature of virtual quantities.

(ii) The Keynesian analysis of national income determination is not static in the usual sense of the word. For static analysis is generally a special case of dynamics. "In defining the term *dynamical*, at least two possibilities suggest themselves. First, it may be defined as a general term including statical as a special rather degenerate case." (SAMUELSON) In this sense, Keynesian analysis is obviously not static.

(iii) Economic quantities are divided into virtual and realized variables. Analysis of realized quantities can be either dynamic or static. Analysis of virtual quantities is different, not in method, but in object. We can nevertheless speak of static analysis of virtual quantities. So long as the virtual character of the variables is safeguarded, analysis of national income determination is both virtual and static. It is static in that each period requires its own analysis.

JOHN MAYNARD KEYNES

A critical analyser of Keynesian thought always diagnoses serious contradictions. The source of the basic lack of logic in the *General Theory*, which shows up in and after Chapter III, lies in failure to distinguish between virtual and realized quantities. In the history of economic thought, KEYNES' genius is none the less responsible for the discovery of virtual factors. In price theory, no virtual factor or force appears. In both dynamic and static analyses, all prices are determined by real factors of supply and demand. But this is not so in the case of national income.

John Maynard KEYNES is not, of course, the first author to compare total supply and total demand. Knut WICKSELL had already proposed his analysis of the two rates of interest, Supply being greater or less than Demand according to the lower or higher position of the real rate of interest relative to the money rate. In the *Treatise on Money*, the proposed theory still followed tradition. In this work, inequalities between savings and investment, that is, between Supply and Demand, are basic. Inequalities $S \neq I$ account for (i) increasing or decreasing national income, (ii) net profits or losses, and (iii) inflation or deflation.

Right after the publication of the *Treatise*, KEYNES, spurred on by his critics, recognized the necessary identity of Supply and Demand, and consequently of savings and investment, since income is defined by purchase expenditures, unspent income being a contradiction in terms. Inequality $Y - (C+I)$, which is the same as $S - D$, is inconceivable. If Y necessarily equals $C+I$, the theory can no longer rely on

$$S \neq D,$$

for at all times

$$S \equiv D.$$

In the same *Treatise*, KEYNES dismissed this identity, which would have given him trouble, by his peculiar definition of national income;

for him national income includes only wages and excludes the firms' profits. "We propose to mean identically the same thing by the three expressions: (1) *the community's money-income*; (2) *the earnings of the factors of production*; and (3) *the cost of production*; and we reserve the term *profits* for the difference between the cost of production of the current output and its actual sale-proceeds, so that profits are not part of the community's income as thus defined." (I, 123)

Identity $Y \equiv C+I$ is thus avoided. Income falls short of spending,

$$Y < C+I,$$

when purchases of producer and consumer goods, having exhausted wages, find an additional source of finance in the expenditures decided by the firms themselves. Thus, the spending of profits, unlike other expenditures, is not taken out of national income, which by definition does not include profits. This procedure is contrived, for it is obviously unusual to eliminate profits from the definition of national income.

Some real inconsistencies followed, of which we shall mention only one. Since savings are part of national income, firms cannot save, says the *Treatise*. By spending profits to purchase consumption goods, a difference between savings and investment immediately shows up. Inequality $Y-(C+I)$ is identical to $S-I$. But, before profits are spent, we have $Y-(C+I)=0$. Expenditure of profits strengthens $C+I$ in relation to Y, and brings about a positive difference between investment and saving: $I-S>0$. But investment does not vary in the process, when profits are spent on consumption goods. Thus the difference between I and S can be attributed only to a decrease in savings. In short, the spending of profits for consumption must lower the level of total saving. Therefore profits cannot be excluded from national income.

In 1931, one year after publication of the two-volume *Treatise on Money*, KEYNES decided to develop a new *Treatise*, which became the *General Theory*. The connection between the two books is regarded differently by different authors. Some, like SAMUELSON, think that the *General Theory* provided a new start. Others, like HARROD, SHACKLE and LEIJONHUFVUD, seem to give some preference to the *Treatise*, since they hold that KEYNES' contribution was not fundamentally renewed in the *General Theory*. In our third chapter, we shall attempt to show that the basic thought is actually the same in both works. For the time being, we should like to point out the main difference. Lines of analysis cannot be strictly the same, since in the *General Theory*, identity $I \equiv S$ replaces adjustment relationship $I \neq S$ which is central in the *Treatise on Money*.

The author's method must be followed attentively.

1° If profits are included in national income, identity $Y \equiv C+I$ is inevitable. And to exclude them is only an expedient.

2° Causal analysis postulates adjustments between Supply Y and Demand $C+I$.

3° When identity replaces any possible inequalities, must we forgo all causal analysis?

4° If profits are not excluded from national income, it would seem that macroeconomic analysis could not possibly be causal.

5° The great contribution of the *General Theory* lies right here. Although total supply and total demand are necessarily equal, they can adjust to each other. The causal character of analysis is thus upheld.

6° The possible variation of Y in relation to $C+I$ is no longer obtained by a truncated definition of national income, but by the usual definition. Macroeconomic income includes income of all holders with no exception.

7° Total supply and total demand lend themselves to causal analysis in a way that neither WICKSELL nor KEYNES (1930) clearly recognized. *Two* factors of Supply and *two* factors of Demand must be differentiated.

8° Realized Supply and Demand furnish causal analysis with moribund forces. To revive them, we would have to go back to the peculiar definitions of the *Treatise* and truncate the definition of national income.

9° Causal analysis rests on an entirely new concept, the *effective demand* of the *General Theory*.

Effective demand is defined at the point of intersection between Supply and Demand, as the *General Theory* explicitly states. However, KEYNES' intuition was too advanced even for him, and throughout the book, he fails to distinguish between Supply and Demand as virtual and realized variables. Yet his innovation will survive in the history of thought: the discovery of the virtual factors of supply and demand. Although the author's logic did not succeed in binding him by the scientific discovery he had just made, this fact, recognized by exegetes, only emphasizes the exceptional originality of his intuition.

"Let Z be the aggregate supply price of the output from employing N men, the relationship between Z and N being written $Z=\Phi(N)$, which can be called the *Aggregate Supply Function*. Similarly, let D be the

proceeds which entrepreneurs expect to receive from the employment of N men, the relationship between D and N being written $D=f(N)$, which can be called the *Aggregate Demand Function.*

"Now if for a given value of N the expected proceeds are greater than the aggregate supply price, *i.e.* if D is greater than Z, there will be an incentive to entrepreneurs to increase employment beyond N and, if necessary, to raise costs by competing with one another for the factors of production, up to the value of N for which Z has become equal to D. Thus the volume of employment is given by the point of intersection between the aggregate demand function and the aggregate supply function; for it is at this point that the entrepreneurs' expectation of profits will be maximized. The value of D at the point of the aggregate demand function, where it is intersected by the aggregate supply function, will be called *the effective demand.* Since this is the substance of the General Theory of Employment, which it will be our object to expound, the succeeding chapters will be largely occupied with examining the various factors upon which these two functions depend." (p. 25)

The 45° diagram illustrates KEYNES' difficult argument.

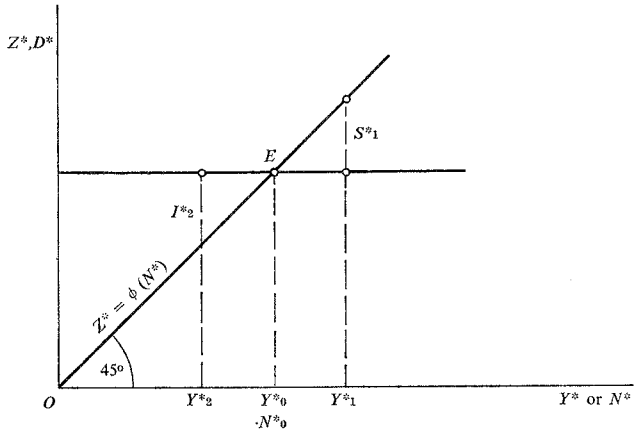

Fig. 27

Assumptions. The number of workers employed, N, is measured in wage units. When wages double in constant money, we say that employment has doubled. However, a definite number of workers is actually employed in period p_n, number N_0 which corresponds to Y^*_0. Other values of N on the x-axis are merely virtual, such as values of Y^* different from Y^*_0. Therefore we should write N^* instead of N. The problem is not one of determining the equilibrium level of employment

in a series where all variables are realized. A single level of employment is realized, all other levels remaining virtual. The equilibrium we are looking for is N^*_0, the only actually observable value among virtual values. Thus, employment N^*_1 corresponding to income Y^*_1 is not observable, since income produced during period p_n is not Y^*_1 but Y^*_0.

Total supply, or income produced in the period, is also measured on the y-axis. There again, a single value of Z^* is both virtual and realized, and all other values are merely virtual. If we measure the supply value of output in wage units, the one-to-one relationships between Z^* and N^* always imply equal terms. Consequently, the curve representing Supply as a function of Employment, $Z^* = \Phi(N^*)$, coincides with the 45° line.

Total demand is also measured in wage units.

Keynes' proof. Demand is called effective at point E which equalizes total supply and total demand. If Supply were greater than Y^*_0, as in Y^*_1,[33] S^*_1 would represent an excess saving, a non-invested saving, which would entail a deficit in sales. Conversely, if total supply were less than Y^*_0, I^*_2 would denote excess investment and therefore an excess demand. Output and employment would then be uselessly curbed, and national income, including firms' profits, would not reach its highest possible level. At equilibrium point E, effective demand is both a demand and an offer. It is supply-demand. "The value of D at the point of the aggregate demand function, where it is intersected by the aggregate supply function, will be called *the effective demand.*"

Logical implications. The determining factors, Z^* and D^*, are virtual. Only one income is produced during period p_n, $Y = Y^*_{0(p_n)}$. All other incomes remain purely virtual. Thus income Y^*_1 is not produced, for, according to firms' forecasts, it would involve a sales deficit. As soon as income is produced, it is inseparably a Supply and a Demand. But determination of national income is none the less possible by adjustment of total supply and total demand. Consider for example the demand expected by firms in connection with period p's output. This constant,[34] known for all values of N^*,[35] is always a virtual quantity, except at point E where it is both virtual and realized. The reason for this is clear. Except at point E, Demand and Supply are separate factors, whereas any realized Demand is only the alias of realized Supply.

Consequences. Interaction between realized Supply and Demand does not affect the level of national income. The determination of the national income to be produced during three successive periods, p_0, p_1, p_2, is shown in three 45° diagrams.

If we transfer the results onto a dynamic graph, we have:

—On the x-axis, periods p_0, p_1, p_2, say 1972, 1973, 1974....

—On the y-axis, the number of workers employed in each period.

1° Each diagram corresponds to only one period.

2° Each diagram indicates the expected demand for each period. Bringing together the two data, time and the level of expected demand, we can represent the dynamic level of national income or employment.

The dominant school is guilty of its greatest logical error in trying to find a link between values E_0, E_1, E_2. They reason as follows. The 1974 income, for example, finds its point of equilibrium at E_2, because this is the only value, in 1974, which is compatible with equality of realized Supply and realized Demand. Suppose we admit that income $Y=E_1$ continues into 1974. According to post-Keynesian analysis, value $Y=E_1$ would elicit by 1974 a positive excess demand which would raise income from E_1 to E_2. But this analysis is completely illogical. If the 1973 income continued through 1974, *no excess demand could appear. For all realized income, Demand necessarily equals Supply.* Therefore determination of E_2 in no way depends on the factors of realized Supply and Demand. To determine E_2, we must return to the 45° diagrams in static analysis. In other words, determination of national income cannot be pushed beyond points E_0, E_1, E_2, of static analysis. The dynamic graph does not represent any time-consuming process. It only reproduces kinematically in a general diagram the successive results of static analysis. In short, the 'income dynamics' is purely descriptive. Analysis is necessarily static, since, in each separate period, it looks to the adjustment of *virtual* Supply with *virtual* Demand.

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The rejection of SAY's law is the "heart of the General Theory of Employment." But this law is extremely long-lived and has not succumbed as easily as was hoped for in 1936. According to the majority School, KEYNES succeeded in splitting total demand from total supply. This prepared the ground for the separate study of supply and demand, as in the analysis of price determination. The error in this viewpoint is the elementary one signalized in our first two chapters.

But the *General Theory*, far from dichotomizing Y and $C+I$, does not write the two terms of the equation as a condition of equilibrium, but as a definitional identity. Since the truncated definitions of the *Treatise on Money* have been dropped, the relationship between Income and Demand is no longer a simple conditional equation. Henceforth we have an identity and not a condition of equilibrium. "Thus *any* set

of definitions which satisfy the above conditions leads to the same conclusion. It is only by denying the validity of one or other of them that the conclusion can be avoided." (*General Theory*, p. 63) The definitions are the following.

"Income = value of output = consumption + investment.

Saving = income — consumption." (p. 63)

The definitions do not threaten SAY's law. Since income is defined as the sum of expenditures, every part of income is necessarily spent. Equality between Income and Expenditure is not conditional, but peremptory. Equality between total supply and total demand is a necessity, as it is in SAY's law.

As soon as Supply is realized, Demand must be brought to its level, which reminds us of the law by which Supply creates its own Demand. In the *Treatise*, this law is trespassed against, since equation $Y=C+I$ is there a condition of equilibrium. But in the *General Theory*, equality $Y=C+I$ is an identity by definition. To invalidate the law of markets, the 1930 definitions would have to be reintroduced, and profits would again have to be arbitrarily excluded from national income. But the truncating of national income is a poor device, abandoned once and for all since 1931. Therefore, SAY's law is reinforced. Not even the slightest disparity is conceivable between output and the demand which it elicits. Quantity demanded is the only means we have of measuring quantity supplied.

However, KEYNES considers the invalidation of SAY's law as his main 'revolutionary' contribution.

"The classical doctrine, on the other hand, which used to be expressed categorically in the statement that 'Supply creates its own Demand' and continues to underlie all orthodox economic theory, involves a special assumption as to the relationship between these two functions. For 'Supply creates its own Demand' must mean that $f(N)$ and $\Phi(N)$ are equal for *all* values of N, *i.e.* for all levels of output and employment; and that when there is an increase in $Z(=\Phi(N))$ corresponding to an increase in N, $D(=f(N))$ necessarily increases by the same amount as Z. The classical theory assumes, in other words, that the aggregate demand price (or proceeds) always accommodates itself to the aggregate supply price; so that, whatever the value of N may be, the proceeds D assume a value equal to the aggregate supply price Z which corresponds to N. That is to say, effective demand, instead of having a unique equilibrium value, is an infinite range of values all equally admissible; and the amount of employment is indeterminate except in so far as the marginal disutility of labour sets an upper limit." (pp. 25-26)

Is this not a flagrant contradiction?

Once profits are included in national income, equation $Y=C+I$ is a definition, an identity. Thus the *General Theory* confirms SAY's law. All saving is invested. Equivalence of saving and investment is a logical necessity.

"Therefore saving = investment." (p. 63) To say that all savings are invested is obviously to deny all possibility of divergence between total supply and total demand.

On the other hand, KEYNES claims to have destroyed SAY's law. Here is found the basic originality of KEYNES' thought.

"Thus SAY's law, that the aggregate demand price of output as a whole is equal to its aggregate supply price for all volumes of output, is equivalent to the proposition that there is no obstacle to full employment. If, however, this is not the true law relating the aggregate demand and supply functions, there is a vitally important chapter of economic theory which remains to be written and without which all discussions concerning the volume of aggregate employment are futile." (p. 26)

It is possible by analysis to resolve the contradiction.

1° *Say's law is valid in the area of realized quantities.*

If we refer to the dynamic graph, we see that all realized income in successive periods necessarily conforms to identity $Y \equiv C+I$, or $S \equiv I$. In real time, every total supply is exactly equalled by the corresponding demand.

2° *Say's law is invalidated in the area of virtual quantities.*

Refer to the graphs of static analysis. In each period, firms must adjust total supply to expected demand. Here we have an equalization process. If, for period p_n's output, firms expect demand $D^*_{0(p_n)}$, they will produce in p_n $Y=D^*_{0(p_n)}$, neither more nor less. Thus if income from full employment is

$$Y_e > D^*_0,$$

a quantum of involuntary unemployment will be unavoidable in p_n.

We can summarize (1°) and (2°) as follows:

1° Realized Supply calls forth realized Demand. No disparity is possible between these two quantities.

2° Virtual Demand governs realized Supply.

The law of markets cannot be invalidated in the area of realized quantities. As soon as production takes place, its corresponding demand must equal it. If spontaneous demand happened to be short, forced

demand would make up for the difference. SAY's law ceases to hold only when transplanted into the area of virtual quantities to which it does not naturally belong. Such was the great discovery of John Maynard KEYNES. Virtual Demand raises Supply to its own level, which is not necessarily the level of full employment.

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In the *Treatise on Money*,

$$Y \not\equiv C+I$$

postulates exclusion of profits from the definition of national income. If profits had not been excluded, identity $Y \equiv C+I$ would have resulted in negating all causal analysis.

In the *General Theory*,

$$Y \equiv C+I$$

is required by definition, the moment that profits are part of national income. So causal analysis is expunged from the area of realized quantities. To allow for an adjustment between supply and demand, analysis must act within a hitherto unknown area, that of virtual factors. Thus we find

$$Y^* \not\equiv C^*+I^*.$$

Like all authors of the dominant school, KEYNES distinguishes only vaguely between Y and Y^*, or between C^*+I^* and $C+I$.

We must emerge from the dark ages of the dominant theories to place KEYNES' originality in its logical framework.

CHAPTER III

NOMINAL MONEY AND REAL MONEY

A. POSITIVE ANALYSIS

1. KEYNES' multiplier theory has much more importance than the dominant school gives it credit for. In equation $\Delta Y = k \Delta A$, where A is the sum of autonomous expenditures, or, more generally, in equation $Y = kA$, kA includes two quantities which should both arouse interest. Theory retains coefficient k, without explaining expenditure A upon which the whole multiplying process rests.

2. As an application of the distinction between realized and virtual quantities, coefficient k has to equal one. The compelling logic of this result cannot be disproved by any experiment.

3. Coefficient $k = 1$ does not weaken the multiplier theory. On the contrary, theory now places its emphasis on A instead of k. KEYNES' multiplier theory implies a revolutionary definition of autonomous expenditures. They must be financed out of money and not out of money income. Macroeconomic theory thus leads to the distinction between nominal money and real money.

IN KEYNES' MULTIPLIER THEORY, COEFFICIENT k IS NECESSARILY EQUAL TO ONE

4. We must distinguish between

$$Y^* = k^* A^*$$

and

$$Y = kA,$$

for virtual and realized quantities must be kept apart as was explained at length in the first two chapters.

THE MULTIPLIER IN THE AREA OF VIRTUAL QUANTITIES, $k^* \equiv 1$

5. What relationship exists between the equilibrium of virtual income, Y^*_0, and the sum of autonomous expenditures A? The definition of autonomous expenditures is not the same for virtual and realized quantities. As virtual quantities, autonomous expenditures define the expected purchases of consumption and investment goods. In the area of virtual quantities, demand is autonomous because it is independent of supply,

$$D^* \not\equiv S^*.$$

6. A definite sum of purchases is foreseen for the output of the period. If A^* is this sum, then the equilibrium of virtual income is

$$Y^*_0 = k^* A^*,$$

where k^* is logically equal to 1. Income which will be produced by firms is equal to expected purchases.

7. If virtual demand suddenly increases by ΔA^*, firms now expect for the same production period a total demand equal to $A^* + \Delta A^*$. The new 'multiplier relationship' is

$$Y^*_0 = k^*(A^* + \Delta A^*),$$

where k^* is logically equal to 1. The rule is always the same. Firms produce income $Y = Y^*_0 = A^* + \Delta A^*$ because they expect demand $A^* + \Delta A^*$.

8. The proof need not be pushed further although we have been taught for years that the multiplier can be different from 1. What gave rise to this misapprehension?

9. A faulty concept of the consumption function underlies the mistake. Authors, not clearly perceiving the difference between virtual and realized variables, rashly assume that the consumption function is valid in both areas.

10. The virtual consumption function is

$$C^* = C^*_0,$$

a single quantum of consumption expenditures being expected for purchase of the period's output. Thus C^*_0 is the sum of expected demands in the sector of the economy which produces consumption goods.

11. The division into sectors is, of course, not accurate. However, if a function is to govern the first sector's output, it can only be $C^* = C^*_0$,

one and only one value of C^* being determined for any period's output.

The sum of autonomous expenditures A^* can be sub-divided into a and cY^{*e}, where a is the necessary consumption and cY^{*e} the 'superfluous' consumption. This distinction is certainly blurred, but let us provisionally suppose that it could be given a precise meaning.

In function $C^*=f(Y^{*e})$, the variable must be written Y^{*e}. Except when constant, virtual consumption cannot be a function of total income: it is exclusively a function of the income produced in the sector of consumption goods.

Equilibrium of virtual income is indeed determined by comparison between the inflow of incomes Y^* and their outflow in final purchases C^*+I^*. All output must recover the money it injects. This is the principle of effective demand. Incomes earned in the sector producing investment goods cannot possibly strengthen demand in the sector of consumption goods. The buyer can certainly spend his income to suit himself. In particular, workers employed in the sector of capital-goods normally spend at least part of their incomes in the first sector. Yet no net money deficit can occur between the two sectors. For the rule is absolute: all sale-deficits must be compensated. Intersectorial transactions take place, obviously, but they necessarily overlap. In other words, inflow and outflow of money is a condition of equilibrium in each sector. For this reason the outflow of incomes spent on consumption goods must be equal to the incomes earned in producing them.

The result is set out in three equations.

$$Y^*=C^*+I^* \tag{1}$$

$$C^*=a+cY^{*e} \tag{2}$$

$$Y^*=a+cY^{*e}+I^* \tag{3}$$

Equation (3) involves no multiplier, since Y^{*e} cannot stand for Y^*.

12. Therefore, we may say that total equilibrium of virtual incomes is found by simply adding up all equilibriums defined in each sector, and even in each industry and firm. If equilibrium of virtual income increases from Y^*_0 to $Y^*_0+\Delta Y^*_0$, we have

$$\Delta Y^*_0=k^*\,\Delta\, I^*_0,$$

where k^* necessarily equals 1.

Multiplier k^* is known in the literature as the 'logical' or 'instantaneous multiplier,' and belongs in the area of virtual quantities, where analysis is necessarily static, since it can refer to only a single period.

THE MULTIPLIER IN THE AREA OF REALIZED QUANTITIES, $k \equiv 1$

13. In current theory, KEYNES' multiplier is a realized and not a virtual quantity. The usual multiplier theory posits a sequence of incomes in time.

14. Since national income is determined from virtual quantities for each period, incomes of successive periods cannot possibly be linked together. The proof of $k \equiv 1$ is therefore a corollary to $k^* \equiv 1$.

15. However, it may be interesting to look for a new method of solution. Let us assume for argument's sake that no proof is available in the area of virtual quantities.

16. If we start from ΔY, two possibilities emerge:
(i) A further income, called induced income, is produced.
(ii) No induced income is yielded. Then we speak of 'leaked' income, or leakages.

17. If, following injection ΔY, i is the coefficient of income induction, the coefficient of leakages is clearly $1 - i$.

18. Both coefficients apply not only to the initial increase ΔY, but also to all induced incomes.

19. The multiplier process must continue as long as the last induced income is still positive. The limit is reached when the whole injection is finally lost in leakages.

20. Now multiplier k can be calculated in relation to induction coefficient i.

If injection is $\Delta Y = 1$, the series of induced incomes goes on until nothing remains of the initial increment. When we can write

$$\text{leakages} = 1$$

the process has come to a stop.

Ratio $\frac{i}{1-i}$ is equal to the ratio of induced incomes to leakages.

Thus, when the sum of leakages equals 1, ratio $\frac{i}{1-i}$ defines the sum of induced incomes. End of multiplier process:

$$\frac{i}{1-i} = \text{sum of induced incomes.}$$

Total income is then equal to $1+\frac{i}{1-i}=\frac{1}{1-i}$. The value of the multiplier follows immediately:

$$k=\frac{1}{1-i}.$$

21. The result of the preceding paragraph does not depend on the nature of the multiplicand. In particular, multiplier k is expressed by $\frac{1}{1-i}$ (i) when the multiplicand is a single injection, and (ii) when it is an injection repeated from period to period.

22. After a single injection, multiplication takes place in a series of successively induced incomes. The sum of induced incomes can be calculated only through time. In this sense, we can speak of a horizontal and cumulative multiplier.

23. If injections are repeated from period to period, when the process terminates the sum total of induced incomes can be measured in a single period. From then on, income stabilizes daily at a value which is a multiple of the daily injection. This is the vertical multiplier.

24. If Y_x is the given income before the initial injection, the increase in income is Y_z. Two identities must be respected,

$$Y_x=(C+I)_x$$

$$Y_z=(C+I)_z.$$

By definition, income is identical to the corresponding demand. To income Y_x corresponds demand $(C+I)_x$ and to income Y_z corresponds demand $(C+I)_z$.

Increment Y_z is *necessarily* spent in purchases, for no fraction of income can be hoarded. An unexpended income is a theoretical monster, for all income is defined by consumption and investment expenditures. Consequently, increment Y_z is entirely spent in purchases. If it were not, a macroeconomic net hoarding would emerge and realized income would no longer equal the corresponding demand. Now, all modern theory agrees that *realized* income is identical to the corresponding demand. In price theory also, *realized* price requires the identity of supply and demand.

25. The fact that increment Y_z is entirely spent gives information about $k=\frac{1}{1-i}$. Coefficient i must conform to this information.

26. The word 'induction' has two distinct meanings.

—Coefficient i_p indicates the induction of *purchases* from incomes; $Y_z \cdot i_p$ gives the value of purchases resulting from injection Y_z.

—Coefficient i_y means induction of *incomes* from purchases; $Y_z \cdot i_y$ defines the value of incomes resulting from the injection.

27. We know already that coefficient i_p equals 1.

$$i_p \equiv 1$$

We use the sign of equivalence because increment Y_z must be wholly spent in purchases. No difference can emerge, since $Y_z \equiv (C+I)_z$ is an identity.

28. Obviously, if incomes can be induced from incomes, induction can only follow from purchases.

29. The hypothesis of a multiplier greater than one requires identity of the two coefficients, $i_y \equiv i_p$.

—If the coefficients are identical, every purchase induces income.

—If the coefficients are distinct, purchases do not induce income and no induction can be produced, since any conceivable induction must occur through purchases.

30. But $i_p \equiv 1$ (§ 24 and § 27). It follows that multiplier $k = \frac{1}{1-i}$ cannot be greater or less than 1, except for value $k = \infty$.

Indeed, if $i_y \not\equiv i_p$, no induction takes place and $k \equiv 1$. For $k \neq 1$, we must have $i_y \equiv i_p$ and, consequently, $i_y \equiv 1$.

31. We have just shown again that incomes do not form a concatenation in time.

32. The theorist must settle between $i_y \equiv i_p$ and $i_y \not\equiv i_p$.

—If $i_y \equiv i_p$, $i_y \equiv 1$ and $k = \infty$. In this case, national income cannot be determinate.

—If $i_y \not\equiv i_p$, $i_y \equiv 0$ and $k = 1$. In this case national income cannot be determined *in time*.

In brief there are two possible pitfalls for the multiplier theory.

—If $i_y \equiv i_p$, income never finds equilibrium, for the initial impulse has endless repercussions.

—If $i_y \neq i_p$, realized income immediately finds its new equilibrium. Consequently, injection is not subject to any dynamic law. Any realized income is as 'equilibrated' as any other.

In the first case, income remains indeterminate, for it cannot possibly be evaluated by any method. In the second case, income cannot be

determined in the area of realized quantities, but it can still be determined in the area of virtual quantities.

—If i_y equals i_p, incomes are linked in successive periods. If these links are not to prevent determination of an equilibrium value, the coefficient of leakages must be different from zero. But we know that $i_y \equiv i_p$ implies $i_y \equiv 1$. Consequently, we are caught in a dilemma. (i) Income must be determined in real time, for successive incomes are connected. (ii) Income cannot be determined in real time, for no leakage is ever produced, so that no law governs the aftermath of any injection.

—If i_y is different from i_p, successive incomes are not connected. Nevertheless, national income can be determined. As a matter of fact, indetermination prevails only in dynamics. But income can be determined *separately* for each period. Thus the two exigencies are made compatible. (i) National income cannot be determined in time, for since $i_y \not\equiv i_p$ involves $i_y \equiv 0$, successive incomes are autonomous with respect to each other. (ii) National income is governed by an economic law. To reconcile (i) and (ii), determination of national income must be sought in the area of virtual quantities.

33. We are already tending toward $i_y \not\equiv i_p$ and therefore $i_y \equiv 0$. But we have not yet quite proved our point.

34. In the area of virtual quantities, we could find direct proof of $i_y \equiv 0$. But in paragraph 15, we set out to find a new solution, which we do find in the idea of injection. Study of the multiplicand is conclusive.

35. Supply and demand are separate factors in the area of virtual quantities. Each is 'autonomous' with respect to the other. To say that expected Demands C^*_0 and I^*_0 are autonomous variables means that they are not identical with income Y^*.

36. However, in the area of realized quantities, total supply and total demand are identical forces. Demand $C+I$ is in no way autonomous with regard to income Y. We must infer, then, that the concept of autonomous spending has no meaning in the area of realized quantities, unless we redefine autonomy to designate something other than the independence of $C+I$ with relation to Y.

37. Actually we can find another definition. We will say that a realized expenditure is autonomous with relation to income if income does not *finance* it.

38. Assume an increase in income Y_z. The holders of Y_z spend it in purchases. To this extent, expenditures are not autonomous, for they are taken out of income.

39. But examine the injection itself. How can income grow from $Y_x = X$ to $Y_{x+z} = X + Z$? We have to posit injection Y_z. Now the injection cannot issue from already existing incomes, for, at most, they are self-generating, and can yield no increment. Expenditure Y_z must be financed in another way, out of a fund distinct from already available incomes.

40. Since $C+I$ is identical to income Y, the injection, or multiplicand, cannot possibly belong to purchase expenditures. Every realized purchase disposes of an already realized income. No purchase, therefore, can define the multiplicand.

*
* *

Paragraph 40 completes our criticism of the multiplier theory. Let us recapitulate.

Consider the following example. Although national income does not vary from instant t_{-x} to instant t, it increases at t and remains constant at its new level until t_x.

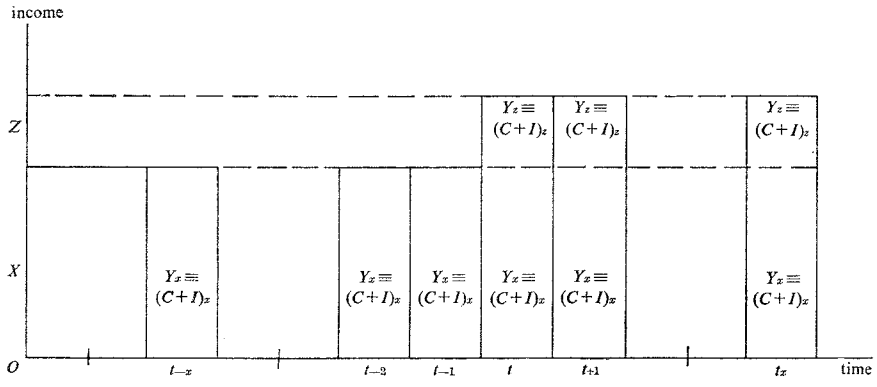


Fig. 28

Each period's income is a realized quantity. And thus it must be entirely spent in purchases. Each period shows the demand which corresponds to the output of the same period. Thus, demand $(C+I)_{tn}$ is the sum of purchases resulting from income Y_{tn} produced in period t_n.

Two hypotheses are now possible.

First hypothesis. Incomes are induced from incomes. Since coefficient i_p indicates the induction of purchases from incomes and coeffi-

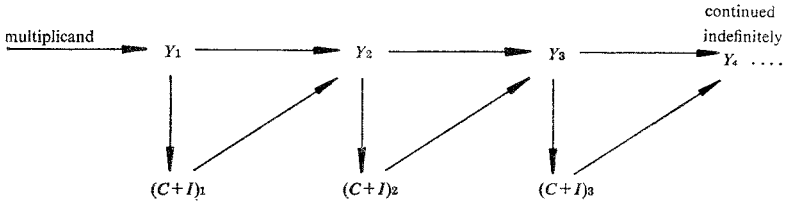
cient i_y the induction of incomes from incomes, the first hypothesis imposes identity

$$i_p \equiv i_y.$$

Second hypothesis. Although purchases are induced from incomes, incomes are not so induced. In this case,

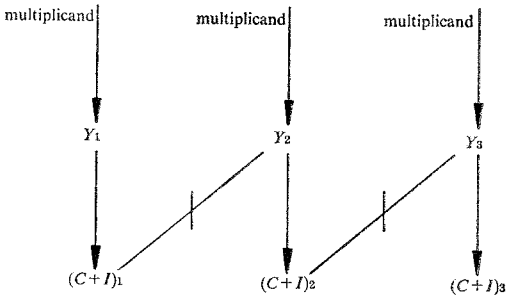
$$i_y \not\equiv i_p.$$

The *first hypothesis* can be represented as follows.



Income remains indeterminate. Each purchase induces incomes and every income induces purchases. Incomes multiply indefinitely through time. A continuous injection would cause an unlimited increase in the dynamic level of national income. The idea of equilibrium value of national income is meaningless here. When coefficient i_y is equal to 1, and it can have no other value according to hypothesis $i_y \equiv i_p$, equilibrium value of national income can never be reached.

The second hypothesis is represented somewhat differently.



—Income and purchases are different facets of the same thing.

—Purchases do not produce income. That is why no arrow points from incomes to incomes. Income of each period is independent of income from other periods. Every income is produced by a multiplicand.

No purchase is a multiplier. The multiplier k equals 1, which is necessary in this hypothesis where $i_p \not\equiv i_y$. If no induced income results from purchases, no induced incomes can be produced at all. Coefficient i_y is then equal to zero, which results in $k \equiv \frac{1}{1-i} \equiv 1$.

According to this second hypothesis, realized income can obviously not be determined in real time. However, determination may take place in another way; and this is where the second hypothesis has the advantage.

We still have to offer formal proof. If hypothesis I was formally watertight, we could not reject it. The final demonstration depends on a basic error in hypothesis I from which hypothesis II would be free. The 'analytical tool' which we need is injection or the multiplicand. *Both* hypotheses in fact recognize the existence of an expenditure which has two characteristics: (1) it results in a newly produced income; (2) it is not taken out of a previously existing income.

Symmetry between two categories of expenditures must be respected:

1° Expenditures which create an income without being induced from a previous income.

2° Expenditures which use up an income without creating a new one.

Hypothesis II conforms to this symmetry, and therein lies its strength. Hypothesis I does not have this symmetry and is thus basically imperfect.

The graph below shows that the two following propositions can each be sustained.

(*a*) All incomes are induced from incomes. (*b*) No income is induced from income.

Case (a). All incomes are induced from incomes. We could write two alternative series,

$$Y \longrightarrow C+I \longrightarrow Y$$

or identically

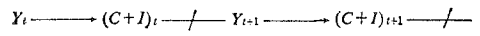
$$C+I \longrightarrow Y \longrightarrow C+I$$

No problem occurs in maintaining the equality of Y and $C+I$. There are exactly as many incomes as expenditures and as many expenditures as incomes. The circuit is closed at both ends. All incomes which appear issue from previous incomes. Therefore there is no entrance. No spent income is lost, for all spent income is recovered in a later income. Therefore, no exit. Not only is the circuit closed at both ends, but, in this case (*a*), the income contained in the circuit travels at even speed. In each period, income of the preceding period is exactly

reproduced. Conversely, any income produced in one period flows out of the previous period. By analogy with physics, no energy can be created and any available energy in Y will be continuously self-generating. Nothing is created and nothing lost.

Case (*a*) is contrary to facts, for national income does not generate itself like a *perpetuum mobile*. Therefore, if the circuit of incomes actually had neither entrance nor exit, we would have to abandon definition $Y=C+I$ and revert, for instance, to equation $MV=PT$, which allows 'injections' in M and a variable speed of money circulation V. But only the formal structure of the case is of interest here. Case (*a*), which will be dropped anyway, is exempt from logical error.

Case (b). No income is induced by income. We can write this in a series of juxtaposed unconnected incomes.



Here again, Y is necessarily equal to $C+I$. Any income is produced by a multiplicand. Every multiplicand is absorbed and destroyed by purchase expenditures. Equilibrium is set up between the creation of incomes (multiplicands) and their destruction (leakages). The circuit is open at both ends. At the entrance, the injection brings about a new income which does not originate in a previous income. At the exit, the leakage annihilates an income which never reappears.

The distinction between cases (*a*) and (*b*) lies in the nature of macroeconomic purchases. Purchases preserve expended income (*a*), or, on the contrary, they destroy it (*b*). If the acquisition of consumption and investment goods maintains the purchasing power of money, no multiplicand is predicated (case (*a*)). All incomes then proceed from purchases. But if the expended purchasing power is lost, the argument rests on the multiplicand (case (*b*)). Since income cannot issue from purchases, it must find some other source, that is, in the multiplicand or injection.

We shall now examine a third case.

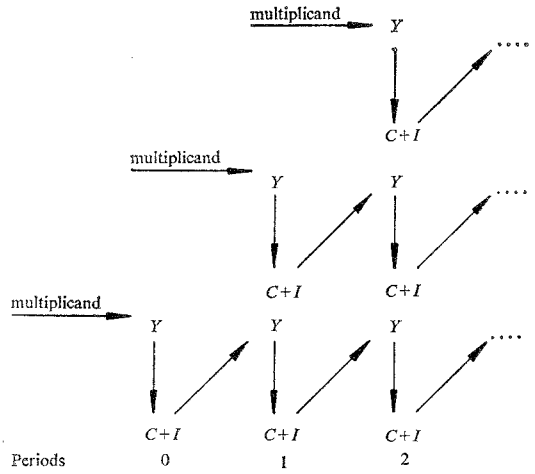
Case (*c*): it is concocted from cases (*a*) and (*b*).

From (*a*). Purchases reproduce income.

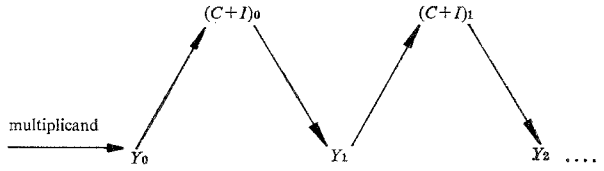
From (*b*). The initial income is not financed by purchases but by the multiplicand.

In case (*c*) we find hypothesis I, since the circuit of incomes is open at its entrance and closed at its exit. The following graph will clear the ground for criticism.

Case (c)



Any period's multiplicand can serve as a starting point. Choose the multiplicand of period 0 and follow the series of incomes which it induces. The simplified graph gives:



The first or initial income comes through the multiplicand and not through purchases. Therefore incomes Y are greater than demands $C+I$. The arrows pointing from Y to $C+I$ and from $C+I$ to Y place $C+I$ in the intervals between successive incomes. Thus the number of Demands equals the number of intervals between Incomes. For N incomes, there are only $N-1$ Demands. This difference will never be made up and will remain significant except when N tends toward infinity. To protect equality $Y=C+I$, we have to choose between (*a*) and (*b*), for case (*c*) is in all finite time incompatible with this equality. *This proves our point. The moment analysis predicates production of income from the multiplicand, it must also recognize the destruction of income in*

purchases. Leakage in the strict sense of irrecoverable loss of expended income parallels the multiplicand.

The following chart sums up the reasoning.

Hypothesis I: $i_p \equiv i_y$.
Hypothesis II: $i_p \not\equiv i_y$.
case (*a*): All incomes are induced from incomes.
case (*b*): No income is induced from incomes.
case (*c*): The multiplicand does not result from already existing incomes, but it originates a chain of induced incomes.

1° The multiplicand fits well into the multiplier theory. No incomes can be induced unless there is an initial injection. Case (*a*) is therefore overruled.

2° Case (*c*) is illogical. It accepts the multiplicand, but not its consequences. To the multiplicand must correspond the leakage, that is, the annihilation of spent income. Final purchases of consumption and investment goods definitively eliminate expended purchasing power.

3° Only case (*b*) remains. The multiplier theory offers to analysis a category of expenditures previously unknown, injections or the multiplicand. If incomes take shape 'initially' through an expenditure which is not an income expenditure, it follows that incomes are spent and not reproduced by purchases; they leak out.

4° Since purchases do not induce incomes, hypothesis I cannot be upheld.

5° Hypothesis II, formally implicit in the multiplier theory, since this theory rests upon the existence of the multiplicand, requires

$$i_p \equiv 1$$

$$i_y \equiv 0.$$

Income created by the multiplicand is spent in purchases. The propensity to spend incomes is equal to one.

But purchases do not induce income. Created income leaks out in final purchases. Incomes feed purchases, and purchases do not feed back incomes.

6° The multiplier theory thus ends in a better understanding of its own starting point, the multiplicand.

In any period, payment of the labor force forms a new income. Income $(C'+I')_{tn}$ is not taken from any previous income. It is entirely a multiplicand. In this connection, it would be useless to try to differentiate between incomes and their increments. In the chart on p. 132, income Y_x is a multiplicand just as income Y_z is. Thus, from t on, the multiplicand is Y_{x+z} and not just Y_z.

Income produced in any period is necessarily spent in purchases of the output which gave rise to it. Purchases being exhausted, income has disappeared once and for all.

Finally, the hypothesis of the dynamic sequence of incomes is entirely destroyed by the very existence of the multiplicand.

The multiplier theory,

$$Y=k \text{ (multiplicand)},$$

becomes the theory of the multiplicand. Coefficient $k=1$ does not diminish the great importance of KEYNES' multiplier theory. On the contrary, the coefficient is exactly known and invariable. The law is thus more exact and strict. Equation

$$Y=\text{multiplicand}$$

states the causal law upon which all macroeconomics is based.

*
* *

41. The two fundamental operations remain to be explained.

Multiplicands. Nominal money is converted into real money.

Leakages. Conversely, real money is finally reduced to nominal money.

NOMINAL MONEY AND REAL MONEY

42. We shall first state a few assumptions.

(i) The economy includes manpower and equipment of all kinds. It produces goods and services intended for consumption and investment.

(ii) At the beginning of production period t_0, for instance, no money is circulating. This didactic assumption brings in no basic distortion.

(iii) Goods and services produced in periods prior to t_0 are already sold out at the beginning of t_0.

(iv) One bank represents the whole banking system.

(v) To simplify analysis, we shall let two firms stand for the entire economy.

(vi) Workers are paid in money and not in real goods.

43. The first step is doubtless the most important. The firms apply to the bank for money to pay their factor costs. In current terms, we say that the bank creates purchasing power which it advances to the firms. This statement is incorrect. The bank creates money, but how do we know that this money has purchasing power? By assuming the identity of money and purchasing power, we miss the point. Theory needs proof that the bank has the faculty to create purchasing power.

44. New money mined like precious metal could perhaps be identified directly with purchasing power.

45. However, modern money is purely fiduciary. It is a paper or book money. Herein lies the problem. Why should something of no real value have power to buy produced goods, services, or assets? 'Monetarists' in a disparaging sense try to base the value of money entirely on confidence. The seller accepts money because he knows others will accept it from him. But this begs the question. The purchasing power which we take over must first be explained. The seller bases his confidence on purchasing power which really exists. But we must not dodge the real problem. Money as it is issued by the bank has no economic value, or very little, for it has no connection, natural or legal, with any commodity.

We shall outline briefly how money is created, although the theory of money lies outside the scope of this work.

46. In creating money, the bank must recognize itself to be the client's debtor. And the client simultaneously becomes the bank's debtor. Money creation is simply the exchange of I.O.U.'s between the bank and the rest of the economy. No economic value can be produced in this manner. The bank itself goes into debt, to the client's benefit. What is the motive behind this debt?

Can the client, or any new holder of a credit on the bank, require reimbursement for this credit? Obviously not. Anyone who would try to get payment for his bank credit would have the same credit returned to him. The bank's I.O.U. is never honored and it is not supposed to be. When credit falls due, the bank note is simply called back and credit is cancelled.

Money creation is thus analyzed as the loan of a debt.

When a bank creates money, it does not really owe anything to its clients. But it pretends to. Whenever a bank manufactures money, it

would be impoverished if the client did not issue an equivalent debt in the bank's favor.[36] As a result, money creation implies that the bank issues a debt on itself balanced by an equal debt taken on by the client. The debt assumed by the bank is not handed out to the client for good; it is only lent to him. When the loan falls due, the promise to pay remains unfulfilled, is taken back by the bank and destroyed.

47. What is the economic value of a book-entry or a paper indicating a debt which will never be paid? If we say that this value is positive, we stick by the old theory according to which money, by nature or by law, has intrinsic value. In olden days currency either consisted in precious metals, or, if fiduciary, it was defined as gold by a legal act. Today analysis must advance to cope with new facts, for now deposits and paper money have no material value, since (i) they are made up of worthless material, paper and ink, (ii) the legal definition of money in terms of gold no longer obtains.

Has money therefore no economic value? It still has its purchasing power, which has always made up its peculiar value. But this purchasing power must be accounted for.

The explanation seemed simple when money was gold or gold-paper. The value of the gold was compared to the value of the collection of commodities. The 'gold content' of money defined its purchasing power relative to the various commodities. If this method were still used, modern money would have no purchasing power over produced goods and services.

48. How can we correlate modern money with real output? There can be no direct connection between the two, since goods and services have material value, whereas money has none.

The greatest error here would be to assume that the problem is solved, and to take the series of money prices for granted, which would enable us to define immediately the purchasing power we are looking for.

The only workable method is to recognize that newly issued money offers no purchasing power over real goods and services. Once money is created, it has still to be converted into incomes. How is this conversion realized? Once we know the answer to this question, it will be clearly apparent that banks issue money, whereas money incomes are created by businesses producing consumption and investment goods. In short, nominal money is issued by banks and real money is created by firms.

49. Payment of factors of production converts nominal money into real money. Workers are paid either in goods or in money. In the first

case, we have an exchange economy. Our economies are monetary in so far as the labor force, from manual to intellectual, is paid in money. Any money paid out for remuneration of labor services is counted as wages.

50. When wages are paid out, the nature of money changes. For instance: two firms each produce ten homogeneous goods. To pay for the services producing these goods, each firm uses $10 newly created by the bank.

Before production takes place, the wages fund is purely nominal. At this stage it would be premature to speak of the purchasing power of money.

But this is not all.

Even after the 20 goods are produced, the $20 making up the wages fund are still purely nominal. Integration has not yet taken place between money and real goods. The mere fact of producing goods and services brings about no correlation between output and the wages fund.

Integration requires an additional process. Only when wages, whether take-home pay or not, are handed out, correlation takes place between incomes and real output.

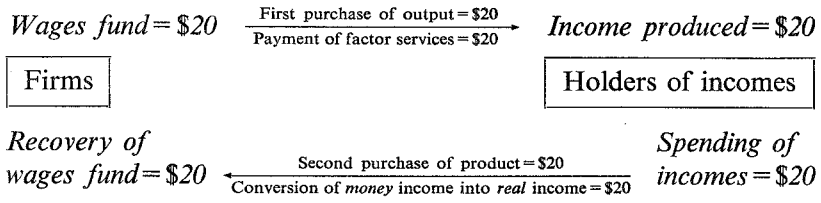
After wages have been paid, money is no longer nominal, but real. This conversion has a precise meaning. The $20 paid out are no longer simple nominal bank money. They carry exact power to purchase the 20 goods up for sale.

Necessary condition. The two firms must recover the $20 paid out to workers, in order to refund the bank.

Sufficient condition. Once the bank is repaid, firms are free from all debt contracted in the process of production.

Thus we see that money is converted into purchasing power by real production.

Diagram representing the traditional theory



The traditional theory may be summed up in a single sentence.

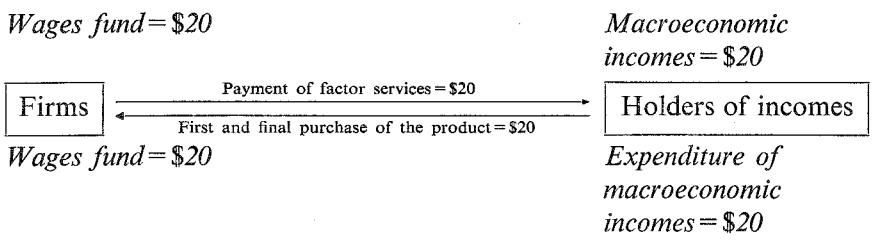
Firms transfer money value to factors of production. Three results follow at once.

(i) In paying for factor services, firms spend a positive purchasing power. They buy what factors produce. The new output is now bought for the first time. According to this doctrine, the wages fund is therefore a money capital which is converted by the firm into real capital, the stock of produced goods. (ii) The labor force does not receive a macroeconomic monetary income. Indeed, money gained by factors is lost by firms. The only net value which emerges is real output. No *money* value is created, all money gains being due to money losses. (iii) If holders of nominal incomes spend them entirely in the purchase of produced goods and services, this expenditure means the repurchase of total production. Here the new output is bought for the second time, since produced goods and services were initially purchased by the firms themselves. The firms' real capital is reconverted into money capital, and wage funds are restored.

The analytical difficulty in this traditional theory lies in the nature of the wages fund. If cash were a commodity in itself (gold or silver) or by legal definition, the payment of factor services could be regarded as the exchange of the newly produced real value for the money value of the wages fund. But, considering the nature of money today, the wages fund is created *ex nihilo* by the bank. Therefore it is not capital or saved income in the true sense of the word. The fund as issued by the bank offers no purchasing power over output. The traditional theory still assumes material money, which has ceased to exist long ago. The real issue is covered up unless we start from a wages fund suitable to modern money, which is at first devoid of any economic value. Bank money has only negligible economic value. The purchasing power of fiat money cannot possibly be manufactured by the bank. Money derives its purchasing power from real output.

Diagram II describes the working of fiduciary money. Firms borrow $20 from the bank. This money is a simple bank credit, inscribed in an account book and/or printed on a piece of paper. The credit issued will be taken back and cancelled when it falls due, since the debt assumed by the bank is never paid. Thus it would be absurd to speak of the economic value of newly created money. However, even though it has no economic value as yet, it can be used as a wages fund, for factors can be paid in nominal money. Modern money is purely fiduciary. It follows that factors cannot be paid in real money. Payment in nominal money does the job. For, as soon as payments are made, nominal money is converted into real money.

Diagram representing the new theory



The substance of the new theory can also be conveyed in one sentence. *Firms do not lose money incomes gained by factors of production.* The three propositions inferred from the traditional theory are thus reversed. (i) By paying for factor services, entrepreneurs spend no purchasing power. They hand out nominal cash, whereas any expenditure of purchasing power is made in real money. Payment of factor services cannot therefore be taken for a first purchase of new output. The wages fund, not being a money capital, is not converted into real capital. (ii) The payment of factors produces a macroeconomic income. Factors gain real money, while firms lose only nominal money. (iii) Expenditure of money incomes by their definitive holders constitutes the first and final purchase of the real output which had initially given rise to these spent incomes.

CANNAN (1927) provides an enlightening analogy, which was taken up by Fritz MACHLUP.[37] Money is compared to baggage checks. CANNAN used this figure to explain the mediating role of the banking system on the financial market. In modern theory, the analogy can have a deeper meaning, for it represents rather well integration between money and real goods. Granting credit, the bank issues notes which will later serve as 'baggage checks.' Firms borrow these notes which they will have to refund three months later, for example. Wage funds thus include notes not yet backed by real output. This is what we understand as nominal money. Before it is exchanged for a real deposit, say a hat or an umbrella, the baggage check is a paper with no value. When firms pay their factors of production with these checks, it cannot be concluded that they buy what these factors have produced. Indeed, the firms buy nothing. They hand out checks which, so far, represent nothing. The bank lends bills, say ten-dollar bills, denominated in the national unit of account. These bills are valuable to firms only because they must refund them when credit expires. It remains none the less true that bills have only the negligible value of the paper they are written

on. This confirms our first result, (i) in paying factor services, firms spend no purchasing power.

But why do factor services accept valueless bills? When put in this way, the question is unanswerable. Factors of production would never work for no pay. However, what is true for firms need not be true for factors of production. Firms, like any baggage clerk, only distribute valueless notes. But the moment these checks are pocketed by factors of production, they take on an entirely new meaning, for they give the right to claim real output. Here again, the analogy is precise, baggage checks have no intrinsic value, but they confer a claim to deposits. Why does fiat money, a simple bank debt, permit its holders to draw upon the nation's output? The answer is straightforward. Firms cannot produce money. Therefore they are forced to recover the bills they paid out to factor services. Thereafter, the purchasing power of these bills cannot be smaller than the real output of the labor force paid in money.

Let us take another look at our example. When factors produce the 20 homogeneous goods they get paid the $20. Suppose the 20 $1-dollar bills do not confer enough power to buy the 20 goods. If this assumption should finally turn out to be untenable, we will have proved our point. Suppose the sale price for 20 goods is $40. Even by spending all its income, the public which holds 20 $1-bills will be able to buy only half the real output. Thus the public is empowered to buy 10 goods and no more. The purchasing power of $20 seems therefore to extend only to 10 goods. But this last inference is faulty. What is true of the public is not true of the bills. Handing over 10 goods for $20, firms themselves become new holders of half the original income. After taking over half the earned incomes, entrepreneurs can spend it in lieu of the first holders. Then the remaining goods are finally bought by the firms. The three charts shown below illustrate: A. The conversion of the wages fund into incomes. B. The twofold result of the public's expenditure of incomes. C. The final reconstruction of the wages fund.

The first chart (A) presents an analytical difficulty. When wages are paid out, the transaction entirely changes meaning according to the standpoint adopted. Wages are paid in nominal money, in 'baggage checks' of no intrinsic value. This is the firms' point of view. However, wages are received in real money, for bills now offer a claim on output from the income holders' point of view. This establishes our second result: (ii) firms do not lose the purchasing power gained by factors. Income is therefore macroeconomic in the exact meaning of the term. The bills represent goods 'deposited' by new production. Firms lose

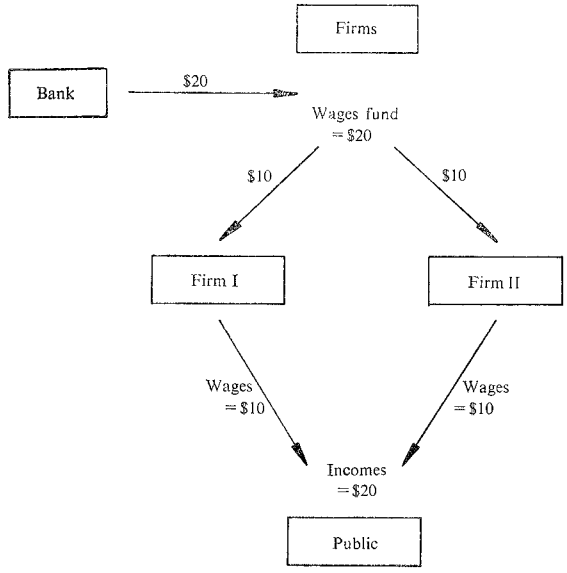


Chart A
Conversion of the wages fund into macroeconomic incomes

no claim to real goods. They only part with nominal checks. Income holders earn a claim that no one loses. Therefore these new claims accrue to society as a whole, no matter who may finally hold them. Bills can change hands many times before all goods are finally claimed.

Lastly, charts B and C confirm the third result: (iii) macroeconomic purchasing power, given by the wages bill, exactly equals the real output of a money economy. And the spending of incomes by their last holders absorbs all the real goods whose production gave rise to these incomes. Incomes in real money can be spent only once. When final purchases have all taken place, no money incomes are left. Once used up, bills retain only their intrinsic and negligible paper value.

The second chart (B) sets forth the following assumptions. The public spends its whole income of $20 to buy goods produced by factors whose wages amount to exactly $20. Firms sell homogeneous goods, at $2 each. Workers paid in money produce 20 goods. All attention must focus on the twofold result of the public's expenditures. When the public spends $2, macroeconomic spending is equal to $1. The other dollar is real money transferred from the public to the firms. Therefore

microeconomic expenditures do not entirely flow into the wages fund. For \$20 spent by the public, only \$10 are recovered by the wages fund. The other \$10 make up the firms' real money income, that is, their profit.

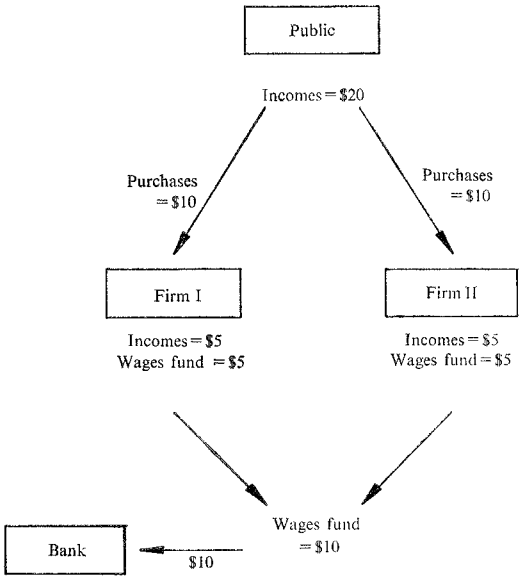


Chart B
How profits are formed

Firms, depositories of newly produced consumption and investment goods, distribute corresponding 'baggage checks.' Clearly, if selling prices are high enough, firms will earn part of the public's claims. Firms sell produced goods and services at a microeconomic price higher than the macroeconomic price equal to the factor cost. In the example chosen, the macroeconomic price of total output is \$20 whereas the microeconomic price of the same output is \$40. By selling at large half their output, firms gain their own income, which gives them power over the other half of their output. A line of demarcation must be drawn between profit and the wages fund. Only when spent will the firms' profits merge with this fund. Right now profits have just been made; they are not yet spent. The spending of profits is shown on the third chart (C).

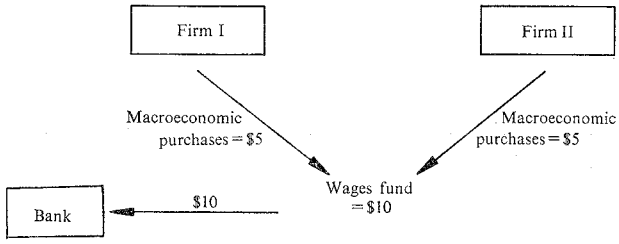
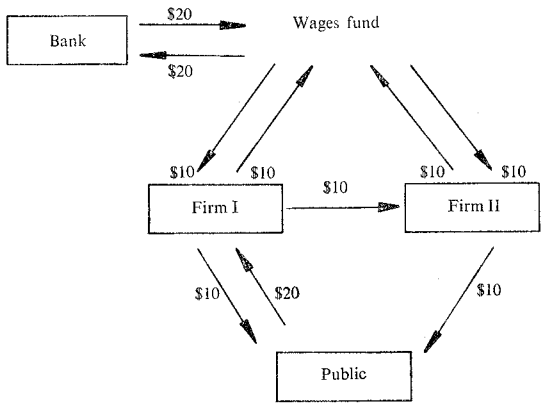


Chart C
Expenditure of profits

Two difficulties remain. (*a*) Firms can buy their own goods only at macroeconomic prices. (*b*) The difference between the earning and the spending of profits is valid even when firms are their own clients.

Point (*a*) can be established immediately. Since profit is a transferred income, a firm cannot possibly produce it directly. Profit must be derived from the public's income.

Point (*b*) shows up better on a general chart.



Firm I sells 10 goods to the public for $20 at a profit of $10. If firm II realizes no profit, the profit of $10 is converted into 10 goods. But the profit of $10 is always converted into 10 goods, even if firm II asks a price higher than $10 for 10 goods. In this case, the profit is simply redistributed between the firms. The one-to-one relationship between dollars and goods is always mandatory. Macroeconomically, one bill goes for one article, and one article for one bill, whoever holds the bills.

The $20 belong first to the labor force and to the state (taxes). In microeconomic sales, the first holders lose half their income to the firms' profit. The public's macroeconomic purchases amount to $10 out of the $20 spent. Against these $10, the public receives 10 goods. For their profit of $10, firms buy the remaining 10 goods. In all cases, the macroeconomic expenditure of a dollar secures the purchase of one real article. When all dollars are finally converted, the wages fund is restored and the bank loan can be repaid.

Summary of paragraph 50.

—Wages are not taken from capital funds. All financial capital originates in savings out of income. The wages fund is revolving and cannot be classed with incomes. It can be compared to a roll of baggage checks which have not yet been offered against a deposit. The wages fund is made up of nominal money, simple bills which as yet have no correlation with output.

—The payment of wages converts the nominal fund into real money. It *integrates* money and real goods. The two integrated quanta are definable in a very precise way.

1° Wages of any kind paid to all factor services in all sectors of the economy, public and private, commercial, agricultural, and industrial.

2° The sum of goods and services produced by factors paid in money. Output need not be homogeneous. Here no 'homogeneity postulate' is called for. When real goods are heterogeneous, they become homogeneous by the very process of integration. Money is a homogeneous commodity and it lends this quality to real goods produced in a money economy.

—Wages give a claim on goods and services produced. The extent of this claim is exactly defined. Wages offer the right to draw the whole output produced by factors paid in money and not in kind.

—However, workers and their families cannot draw the whole social product since we must take account of profits. This could easily lead to confusion. Workers and, in general, the public's real drawing rights are constrained whenever microeconomic prices stand above factor costs. Wages can buy the whole product, but wage owners cannot. With every purchase, part of the spent incomes may be transferred to the firms' benefit.

—The baggage check image explains the process. Factors 'deposit' the output of their work with firms. Against this deposit of produced goods and services,[38] they receive checks conferring the right to claim

the real deposits. However, the first holders of these purchase vouchers are not necessarily their last holders. A redistribution takes place, as a result of microeconomic prices which include transfers.

—When all checks are finally converted by holders, public, state and firms, the whole output initially deposited in firms is withdrawn and appropriated for consumption and investment.

—Only a new output will be able to create new money incomes. Old incomes have vanished by conversion of real money into nominal money. The analogy with baggage checks is sound. Once they have been 'cashed' in, baggage checks have lost all their claiming power.

51. We can now relax the assumptions laid down in paragraph 42. Assumptions (i), (iv), (v) and (vi) are innocuous. We must show that invalidating assumptions (ii) and (iii) does not basically alter anything.

We might think that conversion of nominal money into real money depends on the firms' obligation to repay bank credit. In that case, the assumption of an initially bloodless economy (ii) would be a condition *sine qua non* of the baggage check theory. Assumption (iii) seems more inoffensive. What does it matter if goods are still stocked in firms when new production is begun? New real money bills are added to previous ones, not yet converted, a fact which requires no basic change in theory.

So we must pay special attention to assumption (ii). Firms have their own circulating or revolving funds. Two inferences seem warranted.

a. Restoring their own circulating funds does not seem as binding to the firms as retrieving funds borrowed from the bank. Firms might conceivably not try to recover their funds after having paid the labor force. When funds are not retrieved in the proceeds of sales, is it still true that money acquires its purchasing power only when paid out to factor services? In this case the purchasing power of money seemingly lies in the wages fund, even before this fund is spent in paying factors.

b. Circulating funds may be spent by firms directly to buy consumption or investment goods. They need not be used to pay the labor force. And if spent for goods, cannot these funds exert a purchasing power over and above that of wages? This would prove that the purchasing power of money is not necessarily created in wage payments.

The results of points (a) and (b) amount to the same thing. If true, they show that the conversion of nominal money into real money depends on such a restrictive condition—the primary withdrawal of all money in circulation—that another more general explanation would have to be found.

But points (a) and (b) cannot be accepted. We must examine two possibilities.

First possibility. Funds actually circulate from wage payments to final sales.

Second possibility. Some firms spend once and for all the whole or part of their own circulating funds on final purchases.

Solution of the first possibility. Since funds travel in a complete circle, it matters little whether they are borrowed or not. The only striking difference lies in the interest accruing from the wages fund. If the fund is issued by a bank, interest must be taken from the firms' profit.

Whether firms own or owe their circulating funds is of no critical importance. What is important is that money describes the whole circle. Economics has still to invent the 'wheel': Adam SMITH spoke of "the great wheel of circulation." Since the conversion of nominal money into real money occurs in a transaction between firms and factor services, the bank's intervention makes no difference. Note the money circuit, from factor payments to final sales. Outflow of money in final sales cannot be greater than inflow of money in wage payments. If sale prices are higher than cost prices, *final* sales are only equal to factor costs. All the difference goes into transfers like profits. It is only from the firms' viewpoint that money travels in a circle. From the factors' point of view, inflow of incomes does not describe a circle, because wages are earned and spent in two distinct transactions.

For the firms, a single process is involved, since money is associated with real output in two complementary ways, the inflow and outflow of income. For factors, inflow and outflow of incomes are not two sides of the same transaction, but two separate transactions. Income is earned by work and spent in purchases. Obviously purchases do not provide income, but they spend it. It is just as obvious that work produces income and does not spend it.

Recognition of the double nature of inflow-outflow analysis according to the viewpoint adopted provides the foundation for the distinction between nominal money and real money.

By way of proof, firms do not lose what factors earn: for firms, once the inflow-outflow is completed, the net gain is nil. In short, the firms' only net gain is real and is obtained by spending profits. But expenditure of total incomes, including profits, cannot bring back to firms more money than they spent on factor payments. No extra profit can accrue from the circuit. However, from the factors' viewpoint, inflows and outflows result from two distinct decisions. First, work produces incomes. *And these incomes are net, uncompensated.* Income is not due to the

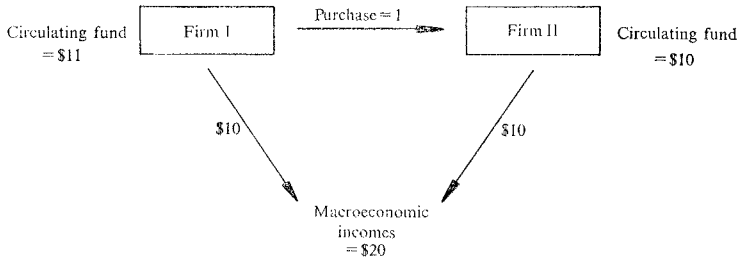
spending of money by firms, but to the spending of effort by workers. Inflow of income cannot be neutralized by its outflow, for inflow and outflow proceed from two autonomous decisions. In the language of logic, it is all a matter of keeping cause and effect separate.

1° *Cause.* From the firms' viewpoint, a single decision is involved. Final purchases neutralize the 'cause,' that is, the production of incomes. Firms give employment, and the corresponding money flows back in final purchases. Inflow and outflow compensate and the net gain derived from the circuit is nil.

2° *Effect.* From the factors' point of view, two decisions are involved. Purchases destroy incomes, that is the 'effect,' the result of production. They in no way thwart the 'cause' or the production of incomes. First income is produced and finally it is spent and destroyed.

Workers earn an income while firms lose nothing. Money must therefore change its nature in changing hands. Firms lose nominal money and workers earn real money. In the circuit from money to output, factor services deposit in firms produced goods and services, whereas firms distribute tickets or vouchers which allow holders to buy the whole output.

Solution of the second possibility. Here, the proof goes somewhat differently. Firms can spend their own circulating fund, or a part of it, to buy produced goods. We must note carefully the definition of circulating funds. Current profits are not included, for they are transferred from the public's income. No analytical problem arises from the fact that firms convert their profits into real goods. The problem lies elsewhere.



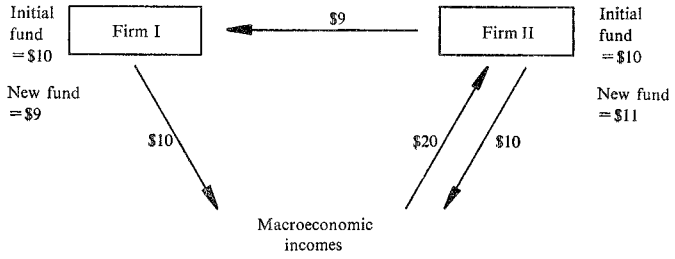
Firm I disposes of a circulating fund not entirely exhausted in paying its labor force. With the rest of its fund, it can purchase goods from firm II, quite apart from the expenditure of profits it may currently earn. Thus it would seem that total purchasing power is made up of two

components: (1) the wages bill; (2) that part of the circulating fund which remains after wages are paid.

However, purchasing power from (2) is not additive to purchasing power from (1). If firm I uses a dollar out of its circulating fund to buy from the other firm, one-twentieth of current production immediately accrues to the firms.[39] We thus ascertain that purchasing power directly spent out of the wages fund finally diminishes factors' real income.

The general rule, then, requires the purchasing power of money to be created exclusively by paying for factor services. The $20 paid to workers can exactly buy the firms' output. If firm I appropriates one-twentieth of the total output, the $20 making up the incomes will be reduced to the purchasing power that $19 had originally. Financed by a circulating fund, purchasing expenditures deplete the buying power of incomes already shared out.

The solution of the second possibility is exactly the same when circulating funds pay for output which is not vendible because it is directly appropriated by firms.



The value of firm I's output is defined by the $10 paid out to workers. But we assume that the firm immediately appropriates one-tenth of its output. This happens when the firm pays wages out of its own fund which it does not intend to replenish. To show up the specific profit realized in the process, we suppose that the firm would realize no profit if it did not alienate part of its wages fund. Any profit earned by firm I will thus be due to the devious usage of a wages fund to make purchases on the commodity market.

Firm II sells its output at a microeconomic price of $20, the corresponding macroeconomic price being $10. From its profit, it takes $9 to buy from firm I all remaining production.

The problem now is: what is the sum of final purchases?

Final purchases from firm II equal $10.

Final purchases from firm I are also \$10, appearances to the contrary.

a. Firm II pays out \$10 for nine-tenths of firm I's output. Thus, *final* purchases of firm II amount to \$9.

b. Firm I realizes a \$1 profit which it converts into one-tenth of its own product. This is consistent with the final expenditure of \$1 from its own fund.

c. Lastly, \$1 is transferred, through the public, from one wages fund to the other. The new funds are \$11 for firm II and \$9 for firm I.

The solution offered bears out the rule already established. But before stating it again, perhaps we should emphasize the foregoing argument. Apparently, firm I realizes no profit, since it pays from its own funds for the purchase of one-tenth of its output.

The truth is not so simple. On the one hand, the firm realizes enough money profit to buy one-tenth of its output. This is proved in *a* and *b*. On the other hand, it loses one unit of its wages fund, a loss which appears in point *c*. The two profits, positive and negative, are logically implied, even if they may be missed by the unobservant analyst.

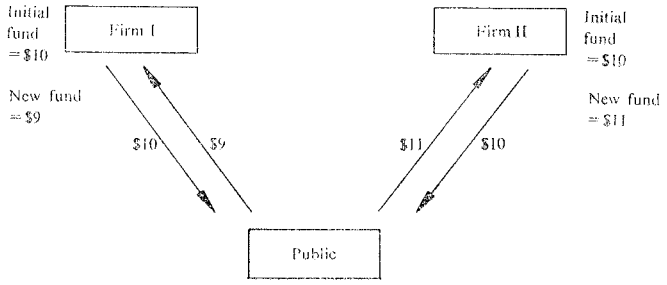
Let us follow the argument step by step. Firm II's profit amounts to \$10. So, if firm II pays \$9 for all available goods from firm I, firm II should be left with a money profit of \$1. This is not the correct result, however. Firm II's profit is completely converted, first into \$9's worth of real goods, and then into increasing its wages fund by \$1. This conversion requires two logical operations. (1) Firm II buys 9 goods for \$10, thus dealing a profit of \$1 to firm I which then buys for its own benefit one-tenth of its output. (2) Firm I transfers one-tenth of its wages fund to firm II.

Final purchases amount to \$20 and these \$20 are entirely spent out of wage-incomes, notwithstanding the fact that firm I spent on purchases \$1 out of its own wages fund.

Assume again that firms make no profits except by purchasing goods directly financed out of their wages fund. Firm I holds on as before to one-tenth of its output. But we now suppose that the public buys the whole (vendible) production from firms I and II. The problem lies in reckoning the sum of final purchases. The correct answer again supports the rule: all purchases must be financed out of wage-incomes.

Analysis requires the distinction between final and gross purchases, or identically between macroeconomic and microeconomic prices. It would be wrong to assert that the public buys for \$9 + \$11 = \$20, and to infer that purchases amount to \$20 for 19 real goods. If this were true, it would again challenge the theory of integration. The sum of purchases,

including the definitive expenditure of $1 from firm I's fund, would then be $21 for 20 goods.



The difficulty lies in spotting the transfer of $1 from one wages fund to the other, for no direct transaction takes place between the firms. But as it happens, the public intercedes. The public buys for $10 firm I's vendible goods. Here a profit is gained in the difference between the microeconomic price, $10, and the macroeconomic price, $9. Firm I then spends this profit to buy its own remaining output. Once spent, the profit is absorbed by the wages fund. Firm I's fund, thus entirely replenished, again equals $10. Then firm I loses to the public $1 from its wages fund. Finally, the public spends $10 — $1 to buy the 9 goods produced by firm I. They also spend $11 to buy the 10 goods produced by firm II. The latter gains the exact amount lost by firm I. *Q.E.D.* The total amount of final purchases, including the final spending of proper funds, is no greater than the wage-incomes handed out to the public. All purchasing power available in the economy has only one origin, namely, wages paid out to factor services.

All purchasing power over any period's output flows from payment of its factors of production.

This law is mandatory under any circumstances. In a money economy the wages fund undergoes two changes. Payment of factor services converts nominal money into real money. Herein originates the purchasing power of money. Then, final purchases change real money back again into a simple nominal fund.

1° *All output elicits just enough purchasing power to absorb it.*

2° *All output is necessarily purchased. If spontaneous purchases are insufficient, the deficit must be made up by forced purchases.*

3° *The purchase of all output absorbs and destroys the buying power to which production gave rise.*

*
* *

Summary.

1° Factors deposit produced goods and services in firms.

2° Firms deposit with factors enough money to withdraw the output.

3° In sales, firms succeed in winning part of the still unconverted real money. When sales are made, part of the money spent by the public is macroeconomically converted into real goods; the rest is merely transferred to the firms which can use it and convert it into real goods to their own advantage. For example two baggage checks are required for the microeconomic purchase of an article deposited against only one baggage check. When selling price is twice as high as factor cost, the public's expenditure really consists of two equal parts. Half of it is converted into real goods and the other half is passed on as real money into the hands of entrepreneurs. Out of the two baggage checks presented for purchase, one is converted and loses all its buying power. This check is no longer real money. But the value of the other check is in no way lessened. Although it has changed hands, it has unchanged purchasing power.

4° If final expenditures made by income holders, including firms, do not spontaneously release all money incomes, firms suffer a loss in nominal money. They can pay for this loss only in real money. The sales deficit thus diminishes the firms' profit, since they are obliged to buy goods nobody wants. The necessary equality between total supply and total demand works at their expense.

5° To avoid all possible sale-deficits, firms try to produce only goods and services which they think will be demanded by income holders. *This is the principle of effective demand.*

6° Baggage checks which have been handed out can be used only once. When goods are bought and carried away, the corresponding purchasing powers are spent and destroyed. This is in accordance with the fundamental law by which KEYNES' multiplier is necessarily equal to 1. Incomes do not form a dynamic chain. No incomes are produced by spending incomes. All factor payments belong to the multiplicand, and every final purchase is a leakage in the sense of KEYNES' multiplier.

7° Identity $Y=C+I$ must be understood in two ways.

(i) Income cannot stand above the corresponding demand. In other words, it is either equal to it, or less,

$$Y \leqq C+I.$$

Baggage checks not yet presented to claim a deposit stand for the new output Y, since they give the right to recover it. Money which has been converted into income must all necessarily be spent in purchases. When spontaneous demand is insufficient, firms lose real money so that total demand finally balances total supply.

(ii) Income must either equal or exceed the corresponding demand. It cannot be less.

$$Y \geqq C+I$$

Purchases cannot exceed available purchasing power. Money income having been created solely by Y, where Y equals all payments due to the labor force, $C+I$ cannot possibly be greater than Y in constant money.

At first sight, demand is greater than the corresponding income: (1) in the case of net dishoarding; (2) in the case of net money creation. But both these cases are inconclusive.

Dishoarding, or negative hoarding, does not exist for society as a whole. In macroeconomics, no aggregate positive hoarding can ever be made. Net hoarding being nil, so is negative hoarding.

Banks create nominal and not real money. If it so happens that newly created money is nevertheless spent on the commodity market, which is conceivable, the purchasing power of wage-incomes is necessarily depleted accordingly. The result of money creation is therefore either neutral or inflationary. If neutral, total demand is not affected. If inflationary, demand exceeds total supply only in current and not in constant money.

B. CRITICAL APPRAISAL OF THE DOMINANT THEORY

In *A Guide to Keynes*, Alvin H. HANSEN presents KEYNES' multiplier theory in its most ingenuous form. Such candor may enliven this study as it approaches its conclusion.

After HANSEN, we shall look again at Paul A. SAMUELSON who seems to deserve the very criticism he directed against John Maynard KEYNES.

Finally, we shall examine in WALRAS' work, and mainly in its modern sequel, with Don PATINKIN, the paradox of the purchasing power of

money. The quantity theory turns in a vicious circle, which can be avoided only by distinguishing between nominal money and real money.

ALVIN H. HANSEN

"Finally we come to Case VI, the extreme opposite of Case V. In Case VI the marginal propensity to consume is unity, as was also true in Case IV. Here, however, we assume, as in Case III, that the initial amount of investment is maintained continuously in each succeeding period. Since the marginal propensity to consume is unity, income in each succeeding period rises cumulatively by the amount of the new continuing investment. Once full employment is reached, this situation would lead to progressive inflation. This case is presented diagrammatically in Fig. 11." (pp. 94-5)

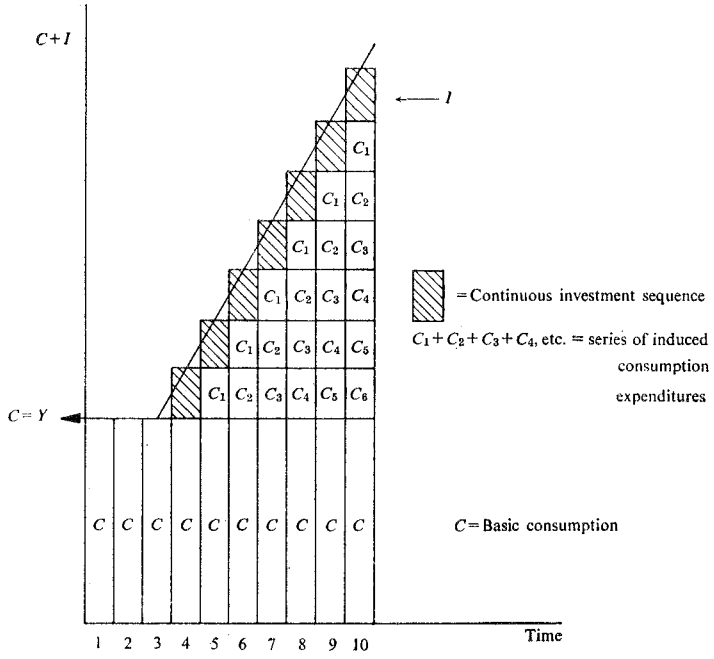


Fig. 11. The multiplier: Case VI.

Fig. 29

1° Why does basic consumption C produce no induced incomes? If the graph were correct, it should show initial consumption in some

remote period. This consumption would correspond to an injection which, unlike investment, would be produced only once.

Figure 29 shows this correctly. The arrow on the graph points toward the past, where we can easily imagine an initial injection occurring in the consumption goods industries.

2° The first unquestionable inaccuracy on the graph can be found in period 4. HANSEN speaks of a "continuous investment sequence." But no injections can be made *via* investment-expenditures, for any investment is a purchase and all purchases are financed out of income. Even inflationary purchases are financed out of incomes, for they diminish other people's real incomes. If injections were defined by investments, we would be implicitly assuming what we had undertaken to explain. No investments can be made out of nonexistent incomes.

3° The error just cited, that is, failure to distinguish between net investments and injections, was frequent in the first stage of the multiplier theory. It perhaps stemmed from the idea that invested incomes are not really spent but saved. In fact, however, the whole output whether consisting of consumption or of investment goods must be bought. The production of real capital does not inject incomes which could flow into the sector of the economy producing consumption goods. The argument runs just as well the other way around. Incomes earned in consumption goods industries are expendable, at least through the financial market, in buying investment goods. Every article produced must be bought. A consumption article can be bought out of incomes earned in producing investment goods. An investment can certainly be paid for with incomes earned by producers of consumption goods.

4° The mistake mentioned in the two preceding paragraphs is better explained by failure to distinguish between realized and virtual quantities.

In the area of *virtual* quantities, Demand is ahead of incomes. The level of expected demands determines the equilibrium level of virtual income. Thus Demand precedes Income.

In the area of *realized* quantities, all realized incomes injected into the economy give rise to equal demands either spontaneous or forced. Now, Income precedes Demand.

Since HANSEN reasons with realized quantities, he has no right to assume a demand which is not already financed by an income. Thus, all the hachured rectangles reflect defective reasoning, for, in figure 29, investments are financed out of nonexistent income.

HANSEN should have used a multiplicand consistent with realized quantities.

5° The hachured rectangles can only stand for injected incomes. But what is the exact definition of injections?

Injection means an income-creating expenditure which is not financed out of previous income. In short, injection creates a new income; it does not reproduce an old one. The difference between injection and induced expenditures is quite clear. The whole series C_1, C_2, C_3,..., includes expenditures financed by incomes, while the expenditure which initiates the series is an injection inasmuch as it is not financed from an already existing income.

6° All this leads up to the most serious inaccuracy in figure 29 which includes injections, but no leakages.

7° This much we can grant: marginal propensity to spend must be equal to one. Any income is necessarily spent and all increments in income likewise induce an equal increase in purchases.

8° The first inconsistency is to assume that induced investments are nil. Net investment goods once produced must be sold. In HANSEN's diagram, the economy produces in each period a new quantum of net investments, but no one ever buys these goods, since all earned income is spent in purchasing consumption goods.

9° The second inconsistency is to assume that a chain of induced expenditures follows each injection. As a matter of fact, each injection can only produce one induction, for the new goods produced must be bought and this can occur only once. To each injection in the sector of investment goods necessarily corresponds an induced purchase of investment goods—a fact of which HANSEN is not cognizant—and a single induced purchase, which HANSEN positively denies.

10° The third inconsistency is to mistake the induction of purchases from incomes for an induction of incomes from incomes. True enough, every injection induces equal purchases, but purchases do not induce incomes.

11° Rid of its main error, the absence of leakages, HANSEN's chart takes the following form.

(i) Injections occur in both sectors of the economy. HANSEN's reasoning implicitly assumes a single initial injection in the consumption goods industries, giving rise before period 4 to a stream of continuously renewed consumption expenditures. This hypothesis of a single injection cannot be maintained. If income produced in the sector of consumption

goods is to remain constant in time, it must be produced by a completely new injection each day.

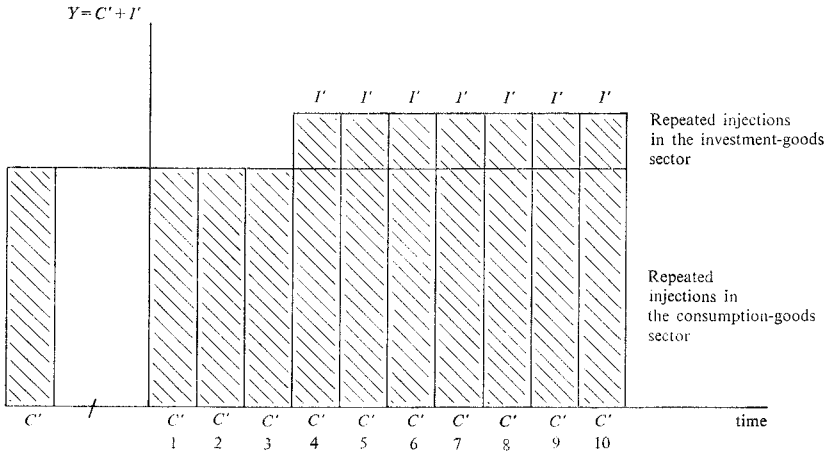


Fig. 30

(ii) Injections could not be fed out of purchases. Whatever the symbol used for it, the multiplicand is not a purchase of produced goods or services, but a payment for factor services. So we write C' and I' for injections made in successive periods.

(iii) Each day, any wage payment is a multiplicand or an injection.

(iv) To say that investment accounts for injection repeated every day is simply to posit a constant income in the sector of capital goods and not, as HANSEN believes, the constant growth of this income.

(v) In fact, since any income created by injection is destroyed by the final purchases induced from it, all incomes produced in any period rest on a new injection which does not augment the level of national income. If the same injection is repeated from period to period, income is constant in time, for these injections only replace incomes destroyed by purchases.

Injections produce real money and purchases do away with it. The same distinction between nominal and real money is required to understand the necessary equality of savings and investment.

PAUL A. SAMUELSON

"Moreover, there is reason to believe that KEYNES' thinking remained fuzzy on one important analytical matter throughout all his days: the relationship between 'identity' and functional (or equilibrium-schedule)

equality; between 'virtual' and observable movements; between causality and concomitance; between tautology and hypothesis. Somewhere, I believe in his early writings, he already falls into the same analytic confusion with respect to the identity of supply and demand for foreign exchange which was later to be his stumbling block with respect to the identity of saving and investment."[41]

Since KEYNES did not understand his own thought, we shall follow SAMUELSON's proof of identity $I \equiv S$.

"Let us assume, however, that I now represents 'intended' investment, and this magnitude equals savings-investment only in equilibrium, i.e., when all the variables take on stationary values. If, however, because of some change, consumption (say) should suddenly increase, national income not having a chance to change, actual savings-investment would fall short of 'intended' investment because of inventory reduction, etc. Consequently, income would tend to rise. Similarly an excess of actual savings-investment over intended investment would tend to make income fall. Mathematically, this hypothesis may be stated as follows: *the rate of change of income is proportional to the difference between intended savings-investment and actual savings-investment.* The discussion here is unrelated to the controversy over the equality of savings and investment despite possible appearances to the contrary. The superficial resemblance between my formulation and the Robertsonian identities whereby the difference between investment and savings is the time difference of income should not mislead the careful reader."[42]

Here, savings-investment, a compound word used to underline the identity, has little in common with Keynesian thought or with the controversies which the necessary equality of saving and investment had triggered after publication of the *General Theory*.

Approached from three different standpoints, the problem of equality $S=I$ becomes more and more difficult to solve.

1° Real goods can first be considered in an exchange economy. Once evaluated, they are divided into two categories, consumption and investment goods. Identity of savings and investment follows immediately. Since all unconsumed goods are saved, investment goods are identical to 'savings goods'. Up to this point, we agree with SAMUELSON. But all the literature which has appeared since 1930 on $S=I$ is inexplicable at this stage. In an exchange economy, savings and investment mean the same thing, that is, all unconsumed riches.

2° If we turn to a money economy, where money does not affect real equilibriums (WALRAS-PATINKIN), saving has two different meanings.

Real meaning. Real savings include all goods except consumption goods.

Monetary meaning. Savings represent money income not spent on consumption goods.

Investment likewise has two acceptations.

Real meaning. Investment goods include all goods except consumption goods.

Monetary meaning. Investment is money income spent on capital goods.

Equality $S=I$ must be examined in accordance with these two meanings.

Equality $S=I$ and the real meaning of the two terms. Refer back to point 1. Savings and investment are synonymous. They denote a commodity which cannot belong to consumption. This is the necessary and sufficient condition defining both a savings and an investment commodity.

Equality $S=I$ and the monetary meaning of the terms. Here, we immediately encounter a real problem.

At first sight, we would conclude that savings do not necessarily equal investment. As a matter of fact, money savings and money investment are not defined in exactly the same way. Savings are the fraction of income not spent on consumption goods. Investment is the fraction of this income spent on investment goods. Thus, a third possibility is open. Hoarding income is to save it; yet apparently it does not generate any investment. Therefore three cases seem possible.

$$S \gtrless I$$

Classical or neoclassical economists rightly reject this conclusion. The assumptions are chosen so that a difference appears between savings and investment. So we assume that the firm produces consumer goods which it offers for money paid out to the labor force. Now these factors hoard, as is their right, all of their money income. Should we not write, then,

$$S>0$$

and

$$I=0?$$

Investment is zero, for no part of the factors' income is spent, whereas investment is an expenditure, a purchase of capital goods. And savings are positive.

The correct reasoning is quite different. The concomitance of a positive income and a zero demand is inconceivable in classical and

neoclassical thought. Factors *demand* the money income which they hoard. If factors bought goods for consumption, savings as well as investment would be nil. But since they do not, investment and savings are both positive. The reason is that firms stock goods *pari passu* with the hoarding of money incomes. Stocks pile up as a consequence of the public's hoarding. The buyers' abstention converts consumable goods into capital goods. Now the firms will be able to produce machines and equipment, even if the corresponding incomes are entirely consumed. Due to hoarding, consumption goods are already available on the market. In short, hoarding creates capital just as investment does. The firms' stock of consumption goods makes up part of the circulating capital, the real wages fund in the RICARDO–SAY–WALRAS analysis. All difference between savings and investment is automatically absorbed in the increase or decrease of the stock of goods accumulated by firms, for 'nature abhors a vacuum.' When an individual demands no commodity, he still demands money and so indirectly he demands commodities. Holders of money incomes really lend capital to the firms.

The whole preceding development is completely ante-Keynesian. With the *General Theory*, necessary equality between savings and investment takes on a new meaning, more difficult to understand.

3° Money is not neutral as the SAY–WALRAS–PATINKIN hypothesis would have it. It is integrated with real equilibriums, which are both monetary and real. This interpretation alone agrees with the Keynesian revolution. Now, equality of savings and investment results from the dual nature of national income, Y being the value of the real supply of current output, and $C+I$ the monetary demand of this output.

SAMUELSON, who misunderstands identity $Y \equiv C+I$, offers an inconsistent explanation of identity $S \equiv I$.

If we compare a rather long passage, which dates from 1941, with some references in later writings, we shall see that SAMUELSON remained convinced of the accuracy of his early reasoning.

"Because of the disproportionate attention paid in the literature to the equality of saving and investment, notice is taken here of PIGOU's complete acceptance of the Keynesian definitions, despite the 'plain' man's wish to regard these magnitudes as not necessarily equal. For the system as a whole, saving and investment, as observables, are defined as the same thing: the difference between income and consumption *when appropriate allowances are made for capital revaluations in the reckoning of income*; or as the value of the increment of capital equipment. For a single individual who can hoard and not consume, and thereby induce

dishoarding and income decreases on others, they need not be the same. From the discussion on page 24, one might get the impression that he attributes the final equality to the fact that individuals cannot on balance increase their holdings of a given total amount of cash. If this is his meaning, it reflects a momentary lapse, since as he himself shows elsewhere (pp. 21–22), the equality of the above definitions holds even if new money is being created per unit of time." (*The Collected..., op. cit.*, "Professor PIGOU's *Employment and Equilibrium*," p. 1184)

Before continuing the quotation from SAMUELSON, we should note that he does not *demonstrate* necessary equality between the positive hoarding of some and the negative hoarding of others. "For a single individual who can hoard and not consume, and thereby induce dishoarding and income decreases on others,..." This rapid sketch recalls Mrs. Joan ROBINSON's reasoning which emphasizes the ingenuousness of the proselytism first engendered by the *General Theory*. "Let us suppose that individuals' desire to save increases—that is, the amount they will save out of a given income goes up—while entrepreneurs are undertaking investment at the same rate as before. Then, some individuals will spend less of their income than formerly. Activity and income in the consumption-good trades will therefore fall off. Owing to this decline in income, consumption will be further curtailed, and a further decline in incomes will take place. One man's expenditure is other men's income, and when one man spends less, other men earn less. As incomes fall the amount that individuals want to save is cut down, and income for the community as a whole is reduced to the level at which the actual rate of saving is no greater than the rate of investment. The more reluctant people are to cut down their saving, the greater will be the fall in incomes." (*Introduction to the Theory of Employment*, Macmillan, 1949, pp. 10–11)

With investment given in period 0, assume that individuals' savings outdistance their investment. Consequently, their income for the following period will be less. Savings will decline. If savings again surpass current investment, income will go on decreasing. Finally, in period p_n, S will again equal I.

Two mistakes immediately stand out.

(i) Savings and investment must be equivalent. "The income derived from producing consumption goods is equal to what is spent on them. Therefore what is saved is equal to the income derived from producing investment goods. In short, the rate of saving is equal to the rate of investment." (p. 7) Now, this equivalence is contradicted in all periods except in period p_n.

(ii) Equivalence must be proved for the sum of the periods. But, in the period from p_0 to p_n, equivalence does not hold.

Like Mrs. ROBINSON, SAMUELSON makes savings and investment identical. He has no right therefore to make income vary in order to adjust S to I. We must monotonously repeat that two equivalent quantities cannot adjust to each other. Now SAMUELSON recognizes this equivalence, taken over from his *Foundations*, where "saving-investment" is a single compound word designating both S and I. "In the above senses, saving and investment may be called for clarity saving-investment. It is not true, however, even in the Keynesian system that for *virtual* displacements, which by their nature cannot be simultaneously observed, the saving (saving-investment) which households *would* perform out of a given income need equal the investment (saving-investment) which entrepreneurs *would* make at that same income. It is precisely because of their being unequal except at one point that income is uniquely determined. The idea of saving and investment being equilibrated, in the sense of schedule intersections, by income and by the other variables of the system, is implicit in the minds of most Keynesians (*e.g.*, HARROD), but is often badly expressed. The concepts *ex ante* and *ex post* attempt to convey the idea, but seem less suitable than the terminology *virtual* and *observable*." (p. 1184) SAMUELSON seems to use here the exact distinction from which the solution results. Virtual quantities of savings and investment are not necessarily equal to each other.

$$S^* \gtrless I^*$$

Equality is necessary only for realized quantities,

$$S \equiv I.$$

But the continuation of the preceding passage brings us to the heart of the misconception. "Equality in the above sense may take time to be established. Until the intersection of schedules defining equilibrium is reached, saving-investment may be inappropriate for both households and enterprises. It is precisely this 'inappropriateness' which acts as the moving force leading a stable system to equilibrium. The problem of time, so carefully worked out by MARSHALL for partial equilibrium markets, is only beginning to receive the attention it deserves in connection with aggregative equilibrium." (*The Collected...*, p. 1184)

The system of Keynesian thought rests on two very clear ideas. (1°) Goods and services must be demanded, otherwise they have no economic value. (2°) Payment of factor services does not constitute a (first) demand of newly produced goods.

1° SAMUELSON readily accepts the first idea, traditional as it is. Even economists who most favor the theory of cost-value recognize that a goods has economic value only if demanded. "If a commodity were in no way useful,—in other words, if it could in no way contribute to our gratification,—it would be destitute of exchangeable value, however scarce it might be, or whatever quantity of labour might be necessary to procure it." (RICARDO, *Principles*, ed. Sraffa, I, 11) A commodity which finds no buyer would be totally useless and of zero value even in a theory explaining value through cost.

2° But SAMUELSON does not catch KEYNES' second idea. If payment of production factors means purchase by firms of goods and services newly produced, the demand of output is by definition equal to its supply. This leads to identity $S \equiv I$ in the SAY–WALRAS meaning and not in the Keynesian meaning.

To distinguish the sum of factor payments, $C'+I'$, from the sum of purchases, $C+I$, it is necessary and sufficient that factors be paid in nominal money. If firms paid out wages in real money, remunerations would be confounded with purchases. But since firms pay wages in nominal money, remunerations are not identical to purchases, for every purchase is an expenditure of real money.

The conversion of nominal money into real money and *vice versa* pushes determination of national income to where it belongs, that is, into the area of virtual factors. SAMUELSON's mistake is to support the theory of the dynamic sequence of incomes. "Equality in the above sense may take time to be established."

Suppose that goods and services produced in any period p_m lead to a spontaneous demand which amounts to less than their factor cost,

$$(C+I)_{\text{spontaneous}} < C'+I'.$$

Here, firms lose nominal money, the money withheld by the public. Only real money can fill the gap. *Filling the gap is the exact meaning of Keynesian dishoarding*. Firms which suffer a sales deficit lose part of their profits in real money. In any period p_m, when buyers hoard firms must dishoard accordingly.

This risk of lessening their sale-profits incites firms to foresee the demand. If $D^*{}_{0_{pm}}$ is the expected demand for period p_m, income produced in p_m will be $Y = D^*{}_{0_{pm}}$. Adjustment between virtual income and expected demand must be started afresh each period. Therefore determination of income cannot possibly be dynamic.

All through his research on national income, SAMUELSON repeated his youthful error. In 1939, he used the 45° diagram in a way which today is commonplace with the majority school. Realized income was thus divorced from corresponding demand, which is absurd.

Again in 1963, SAMUELSON, with the whole dominant school, admits the existence of a link between the consumption function and the level of national income. "One of the important reasons why so many economists went wrong in the notorious prediction of substantial postwar unemployment was their failure to adjust their consumption functions for (*a*) the upward shift from the passage of time itself; (*b*) the war accumulation of liquid wealth; (*c*) the accumulated backlog of needs and desires; and (*d*) the lower propensity to save out of permanently higher incomes." (*The Collected...*, p. 1540)

The doctrine does not seem to have varied since the *Foundations*. From 1937 on, SAMUELSON recognizes the identity of savings and investment. He differs from KEYNES only in the definition of investment. He is not sure that investment should include variations in the value of accumulated capital. But essentially the two authors agree. This point being granted, SAMUELSON himself proceeds to draw the right inference. "Being such an identity and not a condition of equilibrium, it cannot possibly help to determine the level of the rate of interest or level of income." (p. 197) [42bis] This should have stopped SAMUELSON from founding the determination of national income on the equalization of S and I. Instead, he gives a curious turn to his argument. Saving-investment is a single reality. But this single reality is now saving and then investment as the argument requires: "... the saving (saving-investment) which households *would* perform out of a given income need equal the investment (saving-investment) which entrepreneurs *would* make at that same income." (p. 1184) [42bis] Two distinctions overlap: (i) distinction between S and I; (ii) distinction between S and I as realized or virtual quantities. Since we know the remedy, we shall leave it at that.

LÉON WALRAS–DON PATINKIN

Even the briefest study of these two authors recalls one of the greatest controversies in economic theory in this century. Is the neoclassical theory dependent upon a vicious circle? "According to this charge, the value of money—that is, the price level—cannot be said to be determined by its marginal utility. For the utility of a given nominal quantity of money depends on its real value, and this cannot itself be known until

the price level has first been determined. Hence in speaking of the marginal utility of money, we would already be implicitly assuming what we had undertaken to explain."[43]

As we follow PATINKIN's argument, we will find out that the circularity charge, although perhaps badly stated by its first exponents, is nevertheless substantially true.

CRITICISM OF WALRAS ACCORDING TO PATINKIN'S THEORY

1° If money is taken at the 'instant' of purchases, its utility coincides with that of the goods purchased. Money then borrows its utility from the goods it buys. "Of particular interest are such writers as WICKSELL, FISHER, and KEYNES in his neoclassical days—each of whom was at pains to state that 'money as such has no utility except what is derived from its exchange-value, that is to say from the utility of the things which it can buy'—and each of whom then proceeded to base his analysis precisely on the advantage and convenience (utility) of holding money *as such.*" (*op. cit.*, p. 574)

2° According to the last words of the above quotation, money can have utility of its own as cash-reserves.

3° WALRAS did not perfectly succeed in integrating money into his general theory of value. "Thus, although WALRAS must definitely be credited with having presented a cash-balance *equation,* he cannot be credited with having presented a cash-balance *theory.*" (p. 549)

4° Except in the case of a money-commodity, only in the fourth edition of the *Eléments* did WALRAS use in monetary theory the concept of marginal utility. "Indeed, he made no use of marginal-utility analysis in his monetary theory except to deal with the case of a money which was also a commodity—and even here only *after* he had first posited the monetary equation. And in the second and third editions of the *Eléments* he did not even use it then." (p. 546)

5° In his preface to the fourth edition, WALRAS claims with obvious satisfaction to have solved the problem of integration. "Let us now see how all this was changed in the fourth edition. Here again WALRAS starts with the case of a fiat paper money. Now, however, for the first time, he analyzes the demand for such a money in terms of marginal utility. In particular, he speaks of the 'service of storage' *("service*

d'approvisionnement") which is provided by this money; describes the utility of these services in terms of ordinary utility functions; and derives the cash-balance equation from the maximization of these functions subject to the budget restraint." (p. 546)

6° WALRAS' theory leaves something to be desired, since it does not succeed in explaining the specific service which cash-reserves render to holders. WALRAS reasons in a world where everything is perfectly foreseen. Consequently, what would be the need of cash-balances? It would be better to give up all cash, to lend it out at interest, and call it in again at the exact date when, according to perfect expectations, it would be needed.

7° Even by toning down this argument, as MARGET does, (*op. cit.*, pp. 548–549), the fact remains that WALRAS never thought of setting up cash-reserves to meet unexpected or emergency payments. "At no point in the protracted development of his theory of the *encaisse désirée*—not in its first formulation in the *Théorie de la monnaie* in 1886, nor in its revision in the *Etudes d'économie politique appliquée* in 1898, nor in its definitive statement in the *Eléments* of 1900—does WALRAS even hint that he is thinking in terms of a monetary reserve." (p. 549)

PATINKIN hereby concludes that WALRAS' monetary theory lacked economic meaning. "In sum, WALRAS was so anxious to force his monetary theory into the mold of his formal utility analysis that he paid insufficient attention to the details necessary to make this analysis economically meaningful." (p. 550)

8° Léon WALRAS' immediate successors contributed nothing to the solution of integration. However, in 1914, an author who should be better known, Karl SCHLESINGER,[44] finally suggested an exact definition of the demand for money.

Payments foreseen with certainty may require cash reserves, for the flow of receipts and the flow of expenditures are not synchronized. But these cash reserves are necessary. The individual must have them. The theory of utility cannot be applied to them, for the holder has no choice.

When expected payments are uncertain, analysis is basically modified. This time a choice is open to the individual. If he invests his cash reserves, he runs a risk. If a payment suddenly becomes necessary, he will have to sell out of his portfolio assets at an unpredictable price. "The element of alternative choices—and hence of utility analysis—enters only with respect to the uncertain payments." (p. 576)

DON PATINKIN'S SOLUTION

1° His solution is simple and therefore all the more powerful. Money is demanded for itself. More exactly, we must distinguish between nominal and real money balances.

—Nominal balances are simply holdings in cash.

—Real balances are defined by the purchasing power of nominal money holdings over real goods.

—To determine real balances, it is enough to know (i) the sum of nominal reserves and (ii) the general price level.

Individuals demand real money reserves. If these are insufficient, individuals are ready to sell real goods to increase their cash holdings. If reserves appear excessive, people prepare to get rid of the surplus in purchases of real goods. The general principle is to adjust available balances to desired balances. When the supply of money is constant, adjustment is achieved by varying the price level. Thus, a general price increase reduces real balances in terms of a given amount of nominal balances.

2° Since SCHLESINGER's contribution, utility analysis can be applied to money. But the demand for money can also be studied by the excess-demand approach. "Actually, it is not at all clear that it is methodologically preferable to take utility functions—as against excess-demand functions—as the point of departure for the theory of demand." (p. 78)

3° Excess demand of money is identical to excess supply of real goods, as a result of the budget restraint.

4° An individual who wishes to increase his money balances must supply more goods than he demands.

5° It does not follow that the demand for money is passive. To demand money is to supply goods, and the supply of goods is always an active operation.

6° In sum, the theory of money balances finally reverts back to the theory of money transactions. Even if we start out to study money as a stock, that is, *via* the real-balance effect, we must end up by examining the inflows and outflows of money which occur by sales and purchases when real balances are adjusted to their desired level. Money balances in themselves offer no material for analysis. They have to be confronted by the holders' desires. The individual measures his desired real balances by his subjective and objective needs. If he finds the reserves inadequate, he must sell more than he buys. Adjustment of his reserves to his desires

takes place in sales and purchases. If he finds his reserves excessive, he must buy more than he sells. In both cases, the real-balance effect operates by means of transactions.

7° In his *Wicksell Lectures*, PATINKIN emphasizes the effect of real money reserves on capital. Even then, he cannot help having his "wealth effect" take place by purchases and sales. Purchase of consumption goods is simply replaced by the purchase of houses and household goods, oddly enough included in the portfolio of assets. "For the direct wealth effect of a monetary increase can manifest itself not only in the demand for current consumption goods, but also in the demand for the goods which make up the individual's portfolio of assets. ... the individual's portfolio consists not only of money and bonds, but also of such physical assets as real estate and consumer's durables."

8° If the PATINKIN effect is to work at all, it must in all cases flow into the market for real goods, whether consumable or durable.

"It is thus essential to make clear at the outset the sense in which 'utility of money' will be used in the present discussion. Clearly, it does not represent MARSHALL's use of this term. Nor is it intended to denote the utility of the money commodity; indeed, we continue to assume a fiat paper money precisely in order to avoid any ambiguity on this score. Nor, finally, is it intended to denote 'the marginal utility of the goods for which the money can be exchanged.' Instead, our concern is with the utility of *holding* money, not with that of *spending* it. This is the concept implicit in all cash-balance approaches to the quantity theory of money; and it is the one that will be followed explicitly here." (*Money*, ..., *op. cit.*, p. 79)

9° But finally, the utility of holding money is identified with the utility of spending or earning it, or, in short, with the utility of buying and selling real goods.

Let us suggest a comparison with river and lake. The flow analysis (river) is 'Keynesian.' Analysis of money stock (the lake) is 'Walrasian.' *But obviously the two approaches are strictly identical.* The lake's level can vary only by the difference between the inflow and the outflow of the river. If our purpose is to study the level of stocked cash, we must study the inflow and the outflow of incomes. All demarcation between these two analyses is purely deceptive.

CRITICISM OF PATINKIN'S SOLUTION. REVIVAL OF HELFFERICH'S CRITICISM.

Assume constant real balances throughout a market session. According to HELFFERICH and PATINKIN, money prices must be known

to start with, otherwise individual behavior could not be observed. HELFFERICH infers from this that money value is a datum and so it cannot be determined on the market. PATINKIN takes exception to this. For him, money value is at the same time a datum and an endogenous variable. Given at the opening of the session, it can be modified during the market. "In brief, here again is a confusion between types of experiments. In a market-experiment, money prices are the variables whose values must be determined. Hence it would truly be a case of *petitio principii* to assume that prices are already determined. But in an individual-experiment, the amounts of excess demands are the variables to be determined, and money prices are the independent variables whose values *must* be given in order to conduct the experiment. Clearly, there is no circularity in stating that the market excess-demand equations derived from such individual-experiments are then used to determine the equilibrium money prices of the market-experiment." (p. 116)

PATINKIN's argument is faultless, *but it is beside the point.* The error lies in deriving the purchasing power of money from the general price level,... "the value of money—that is, the price level." (p. 115)

Knowing previous prices, individuals declare their excess demands and supplies. New prices result from these excess demands. But price variation leaves the purchasing power unaltered, appearances notwithstanding.

On a pharmacist's balance, two money units at the beginning of the experiment counterpoise one unit of commodity *b*. The two coins pay for one *b*. But an excess demand may appear during a new market session. If so, the price level must normally increase. Assume that the new equilibrium requires 3 coins for the purchase of one unit of *b*.

Two results follow, one a tautology and the other an error in logic.

Tautological result. If the new exchange price is 3 coins for one unit of *b*, one coin buys $1/3$ *b*. We can speak of the price of the money in terms of *b* just as we can speak of the price of *b* in terms of money. "There is no reason why we should not enjoy the semantic liberty of saying that goods buy money and of describing, accordingly, a demand function for money. For if such a function is correctly described, it cannot but be the obverse image of the aggregate demand function for goods; hence, as we shall always be at pains to show, the side of the transaction from which the analysis is conducted cannot affect the conclusions reached." (pp. xxiii–xxiv) Whether we say that 3 units of money pay for 1 *b*, or 1 *b* pays for 3 units of money, we are saying the same thing, one way or the other. Why should PATINKIN feel obliged to defend such a truism, "as we shall be at pains to show?" He thinks he

is offering positive information about money demand, and, consequently about money value.

Illogical result. On our pharmacist's balance, let us put the money on one side, money itself and not "the value of the money" whose definition we are looking for.

True, an abstract unit of account has no physical existence. "Though we have spoken of 'money' in the singular, we shall formally assume the presence of two distinct types of money in our economy. First, there is an abstract unit of account, which serves only for purposes of computation and record keeping. This unit has no physical existence; that is, it does not coincide with any of the goods which exist in the economy. Examples of such money in various societies are well known. Perhaps the most familiar is the guinea in present-day England." (p. 15) We have no intention of criticizing at length the slight confusion in this statement, for the English guinea actually existed and was equal to 21 shillings. On the other hand, why look afar for what every country has right at home? Any country's money is both a unit of account and a unit of payment. Any bank money is, first, a unit of account or a collection of such units, for it bears a denomination and a number. Second, it is a unit of payment or a collection of such transferable units. We can infer, then, that any existing amount of money is at one and the same time a sum of units of payment and of units of account. Conversely, merely to define a unit of account does not constitue money.

To return to our balance: money placed on one side is both a unit of account and a unit of payment. This means that the real goods placed on the other side of the balance is measured and counted in the money unit. Since money buys real goods, and real goods do not buy money, the value of produce is measured in money, but the value of money is not measured in produce. The value of money is not determined, therefore, on the market for goods.

Don PATINKIN's conclusion is just the opposite. "Money buys goods, and goods do not buy money. The natural place, then, to study the workings of monetary forces is directly in the markets for goods. This will be our central theme." (p. xxiii) A physicist, for example, uses a pound for a unit of measure. In his first experiment, he finds that two pounds balance one unit of an object *b*. Then, in a later experiment, he needs three pounds to balance one *b*. PATINKIN concludes from this that the pound has changed weight, for it no longer weighs $1/2\ b$ but $1/3\ b$. Obviously, the pound weight has remained unchanged. Only *b* changed weight from 2 pounds to 3. On the market for goods, the law of supply and demand makes the exchange value of real goods

vary in money terms. It leaves money unchanged. Money is the unit necessary for measuring price variations of real goods.

The formal proof of this proposition may be offered in a very simple way. Excess demand for one real commodity (consumption or investment) furnishes one adjustment factor and not two. In the example of commodity *b*, excess demand is cancelled by the increase, positive, negative, or zero, of the money price of *b*. Only one adjustment factor is operative, the equalization of the supply and demand of *b*. Now when only *one* adjustment factor is available, it cannot possibly yield *two* determinations. Only the exchangeable value of *b* is modified, while the purchasing value of the money is not affected in the process.

PATINKIN offers integration only through the looking-glass. Excess demands on commodity and bond markets cause variations in the price of goods. PATINKIN wrongly concludes that money value varies in terms of these goods. Thus integration would be a simple matter; money value is determined at the same time and by the same factors as the value of real goods. It is obvious that such an integration is schizophrenic. The integration of monetary theory and value theory is not too difficult if one is but a mirror-image of the other. "The alternative approach developed in this book begins with a description of the demand functions for commodities and bonds, with particular emphasis on the relatively neglected influence of money balances. These functions are then used to carry out a static and dynamic analysis of the central problems of monetary theory—the effects of changes in the quantity of money and shifts in liquidity preference on interest, prices and employment. In this way, we achieve an integration of monetary theory and value theory: the propositions of both theories are derived by applying the same analytical techniques to the same demand functions of the same markets. Such an integration is desirable, not only for showing these two theories to be special cases of a general theory of price, but also for enabling a simple and direct treatment of otherwise complicated problems." (p. xxiv)

The dictionary says: "A truism is commonplace tautology. The *petitio principii*, or vicious circle, is the fallacy of assuming in the premise of an argument the conclusion which is to be proved."

First, PATINKIN's analysis is a truism. If 3 money units are exchanged for one unit of commodity *b*, it is true, but tautological, to say that one unit of *b* is exchanged for 3 money units.

Secondly, PATINKIN's analysis turns into a *petitio principii*. This time the author tries to prove what he had already assumed. The fol-

lowing text is supposed to break HELFFERICH's circularity charge. "In a market-experiment, money prices are the variables whose values must be determined. Hence it would truly be a case of *petitio principii* to assume that prices are already determined. But in an individual-experiment, the amounts of excess demands are the variables whose values *must* be given in order to conduct the experiment. Clearly, there is no circularity in stating that the market excess-demand equations derived from such individual-experiments are then used to determine the equilibrium money prices of the market-experiment." This passage does not concern HELFFERICH's circle. Money prices must be assumed as a datum if supply and demand curves are to be drawn. The intersection of the curves determines the new equilibrium price. PATINKIN is right: price determination is not circular. Before measuring demand and supply, prices must be assumed. Nevertheless, the new equilibrium price is not given but determined in the experiment. No economist can ever have been unaware of this elementary fact labored by PATINKIN.

The circle to guard against lies elsewhere. *When supply and demand of real goods are newly adjusted, the purchasing power of money remains exactly equal to what it was before these adjustments.* Individual and market experiments determine the money prices of goods, but they in no way alter the macroeconomic value of money.

PATINKIN quotes a criticism which MARX directs against the quantity theory of money. The validity of this theory depends "... on the absurd hypothesis that commodities are without a price, and money without a value, when they first enter into circulation, and that, once in the circulation, an aliquot part of the medley of commodities is exchanged for an aliquot part of the heap of precious metals." "The obvious answer to this criticism is that it is based on a fundamental misunderstanding of the equilibrating process: that commodities are always 'with' a price and money always 'with' a value, but that the dynamic workings of the real-balance effect assure that this price level—or this value—will not continue to prevail unless it is proportionate to the quantity of money." (p. 582) MARX's criticism is right and PATINKIN attacks it in vain. The real-balance effect, as we have shown, operates on the markets for goods. PATINKIN obviously knows this. "Some examples will help clarify this basic relationship. Consider the simple case in which, in the light of the conditions confronting him, the individual plans to retain the initial quantities of all commodities but one. That is, the excess amount demanded of every commodity but one is zero. Assume that this one amount of excess demand is positive. Clearly, the only way in which this additional quantity can be purchased is by drawing down

initial cash balances to the extent of its money value. Thus when the individual formulates the preceding market plan for commodities, he is simultaneously formulating a plan for reducing his initial holdings of money. No additional independent decision on this score is involved." (pp. 24-25) No statement could be clearer. If the individual decides to buy an article, he decides simultaneously to take out of his balances enough money to pay for it, as gifts and theft are not considered in PATINKIN's analysis. Conversely, selling increases the money balance. In other words, all decisions made about money balances—to increase, maintain or decrease them—only mirror decisions made on the market of real goods. PATINKIN thus proves that only real forces are at work in his system. After the price of real goods is determined, no forces remain available to determine the value of money. Therefore PATINKIN only plays with words. He does not really account for the variation of money value. Only the money price of real goods varies as a result of his real or wealth effect. *If money has a value, it is unexplained.*

PATINKIN is not aware of the essential inadequacy of WALRAS' monetary theory. Perhaps "services d'approvisionnement" do not easily fit into marginal analysis. But a much more important question relates to the analytical transition from commodity-money to fiduciary money.

The loss of the equation between supply and demand of the money-commodity creates a void which cannot be filled in WALRAS–PATINKIN's system. If any commodity is chosen as money, it becomes scarce and its value as a real commodity increases. Two questions are thus answered simultaneously: (i) Money value, like that of any commodity, is determined by supply and demand. (ii) The value of all other goods is measurable in money, the general medium of exchange.

We can take a simple example. The economy comprises three commodities, *a*, *b*, *c*. In addition, it uses money for transactions.

First hypothesis. When money is a commodity, one of the three commodities *a*, *b*, or *c*, must be chosen as a means of circulation. Choose *a*. We must distinguish between a_2, the commodity as money in general circulation, and a_1, the same commodity available for consumption and investment. Now comes the problem of determining the value or the price of the four commodities, a_1, a_2, *b* and *c*. Commodities a_1, *b* and *c* are supplied and demanded through the medium of money and their price straightforwardly results from the law of supply and demand. For the remaining commodity, a_2, equation 'unit price of a_2 = unit price of a_1' must be verified, since the price of a_1 takes into account all the uses of commodity *a*, including its monetary use.

Second hypothesis. The money is fiduciary. So we must add m, money. We have only three independent equations, equalization of supplies and demands for commodities a, b and c. One crucial equation is lacking, the equalizing of supply and demand of money. PATINKIN relies on "semantic liberty" to solve the dilemma. Thus, even though we have no equation determining the value of fiat money, we only have to use the very same equations which determine the price of real goods. This reasoning is far too ingenuous. The same factors of supply and demand cannot be used in the same markets to determine both the money value of the goods and the real value of money.

Certain economists had the idea of providing the missing equation in the form of $MV=PT$. We could go on and on talking about this equation, which is fundamentally ambiguous, having two meanings. Consider each separately.

First meaning of $MV=PT$. The letters represent the stock of money M, the stock of real goods up for sale T, and the money price of T. In that case, the equation is absurd. The price of all the commodities together would be equal to the stock of available money, multiplied by a factor V which could be either endogenous or exogenous for all we know. Why would real goods have for their total price a bulk of nominal money, a stock of fiduciary currency notes whose creation has nothing to do with the production of real goods? We would have to assume that the two physical quanta, that is, money balances and goods available for sale, suddenly establish neighborly relations of whose *raison d'être,* nature, and degree we know nothing. In the first meaning, the quantity theory only clogs up the void in the neoclassical system.

Second meaning of $MV=PT$. The symbols stand for the flow of money spent in purchases, M, and the flow of real goods given in exchange, T. If the money flows V times on the average, the total value in exchange of goods bought, PT, is by definition equal to the money spent MV. We recognize the equation of exchange in its true meaning, which is purely tautological.

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* *

Like all economists of the dominant school, Don PATINKIN fails to recognize the distinction between nominal and real money.

Money and goods are not integrated on the market for real commodities

If demand of a 'normal' commodity is greater than its supply, the price of this commodity must increase. *No change of macroeconomic purchasing power of money occurs thereby.*

Banks create no purchasing power

There are no exceptions to this rule.

1° If newly created money is directly spent in purchases, the purchasing power of the money unit diminishes. But even then, this does not result from the real-balance effect. Nominal money mixes with real money.

2° If new money is added to the circulating fund, no effect is produced on the purchasing power of money already in circulation.

BRIEF SUMMARY OF THE ARGUMENT

CHAPTER I. THE LOGICAL ERROR IN THE CURRENT INTERPRETATION OF DEFINITION $Y=C+I$

The mistaken logic stands out with greater clarity if we start with ROBERTSON's dynamic analysis.

A. Identity $Y=C+I$ cannot be broken by "time-lagged" analysis.

B. It follows that identity $Y=C+I$ holds at any moment of real time.

C. The dominant school is inconsistent in its interpretation of $Y=C+I$. The most striking example of this inconsistency is to be found in SAMUELSON's analysis of Keynesian income determination.

A. WHY ROBERTSONIAN DYNAMICS IS LOGICALLY UNTENABLE

1° ROBERTSON accepts definition $Y=C+I$. "Evidently, if we define our terms suitably (and this involves, among other things, counting increments of circulating capital as investment goods), this statement is true, because it is what is called a tautology." (*Money*, Cambridge Economic Handbook series, p. 207).

2° Not only does ROBERTSON accept definition $Y=C+I$, but he was one of its main discoverers. After the *Treatise on Money* was published, Dennis ROBERTSON directed a sharp attack against the exclusion of profits from the definition of national income. "... 'Savings' can only be made out of 'incomes' so that if an entrepreneur spends his 'profits' on the purchase of new machines, he is not 'saving', while if he refrains from spending on consumption a normal income which he has never received, he is deemed to be 'saving'." ("Mr. KEYNES' Theory of Money," *The Economic Journal*, 1931, p. 406) As soon as profits are recognised as part of the community's income, identity $Y=C+I$ follows inevitably.

3° Yet ROBERTSON is anxious to retain equation $Y=C+I$ as a condition of equilibrium. "On my definition, Saving and Investment are *not* necessarily equal, and it is on the difference between them that the *movement* of the price-level (...) depends." ("Saving and Hoarding" in *Essays in Monetary Theory*, London, 1948, p. 93)

4° How can ROBERTSON reconcile the two apparently contradictory statements?

(1) $Y=C+I$: definition

(2) $Y=C+I$: condition of equilibrium

He relies on his "step by step" analysis. "I assume the existence of a period of time, to be called a 'day', which is finite but nevertheless so short that the income which a man receives on a given day cannot be allocated during its course to any particular use. A man's disposable income—the income about which the question arises on any particular day as to whether it shall be 'saved' or 'spent'—is thus the income received not on that day but on the previous one. A man is said to be *saving* if he spends on consumption less than his disposable income." ("Saving and Hoarding," in *Essays*, p. 65)

5° Write Y_t^{produced} for income produced in period t and D_t for total demand in the same period.

ROBERTSON's definitions are

$$D_{t+1}=f(Y_t)$$

Income produced in period t is not available in period t, so that the corresponding demand is D_{t+1} and not D_t. Demand is a function of the preceding income.

6° ROBERTSON's analysis proceeds as follows.

Proposition 1. Income produced in period t is not available before period $t+1$.

This proposition need not be proved. It is a matter of pure definition. More precisely, it merely establishes the meaning ROBERTSON freely gives to the words he uses, like income, expenditure, and savings.

Proposition 2. In period $t+1$ total demand may be greater or smaller than available income.

The second proposition is an inference. It either follows or does not logically follow from ROBERTSON's definition. We shall presently demonstrate that proposition 2 is indeed nothing but a *non sequitur*.

7° Let us first proceed with Dennis ROBERTSON.

Case I.

$D_{t+1} > Y_t^{\text{produced}}$: dynamic increase in the general price level or in the level of national income.

Case II.

$D_{t+1} < Y_t^{\text{produced}}$: dynamic decrease in the general price level or in the level of national income.

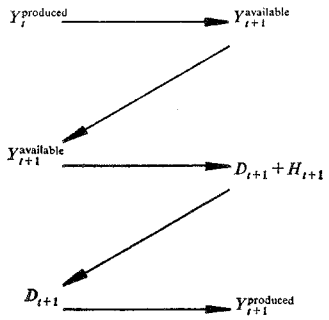
Case III.

$D_{t+1} = Y_t^{\text{produced}}$: dynamic stability in the general price level or in the level of national income.

Income increases when new money is injected by the banks, or when old incomes are dishoarded in the current period.

National income decreases when banks withdraw part of the money circulation, or when a positive fraction of current incomes is put aside into hoards.

8° *Robertsonian sequence.*



The income produced in period t becomes available in period $t+1$. Disposable income in period $t+1$ is spent (D_{t+1}) or hoarded (H_{t+1}). Only the fraction of disposable income which is spent, that is D_{t+1}, yields the new income in period $t+1$. Thus income decreases when money is destroyed or hoarded, and it increases when money is created or dishoarded. National income remains stable between t and $t+1$ when the whole of disposable income and nothing more is spent during period $t+1$.

9° In the following chart, it is easy to perceive ROBERTSON's mistake at one glance.

Robertsonian sequence *A non sequitur:*	*Correct inference:*
$D_{t+1} \gtreqless Y_t^{\text{produced}}$	$D_{t+1} \equiv Y_t^{\text{produced}}$
$D_{t+1} \equiv Y_{t+1}^{\text{produced}}$	$D_{t+1} \not\equiv Y_{t+1}^{\text{produced}}$
$Y_{t+1}^{\text{produced}} \gtreqless Y_t^{\text{produced}}$	$Y_{t+1}^{\text{produced}} \gtreqless Y_t^{\text{produced}}$

ROBERTSON starts from identity $Y=C+I$, which he calls a tautology. Now, he is quite free to introduce lags into his analysis. These lags allow him to write $Y_t \neq (C+I)_t = D_t$. But identity $Y_t = D_{t+1}$ could only be dismissed if identity $Y=C+I$ did not hold, which is absurd, for $Y=C+I$ is ROBERTSON's very starting point. If $Y \neq C+I$ in the first place, no dynamics would be called for. The demand corresponding to Y_t^{produced} must be equal to Y_t^{produced}. Now, given the Robertsonian "day", the total demand which corresponds to the income produced in period t is D_{t+1}. As a result, identity $D_{t+1} = Y_t^{\text{produced}}$ is nothing but the definitional identity $Y=C+I$.

One objection remains open. Does not identity $D_{t+1} = Y_{t+1}^{\text{produced}}$ suffice to corroborate identity $Y=C+I$? *It does not.* If we write

$$D_{t+1} = Y_{t+1}^{\text{produced}}$$

and

$$D_{t+1} \neq Y_t^{\text{produced}}$$

the definitional identity $Y=C+I$ is flouted.

The logic of this conclusion is quite clear. If D_{t+1} is smaller than Y_t^{produced}, identity $D_{t+1} = Y_{t+1}^{\text{produced}}$ means that all expenditures out of the disposable income yield a new income, although the disposable income is not wholly spent. In that case, all D is Y, but all Y is not D. It then follows that $D < Y$, which flatly contradicts identity $Y=C+I$. The inconsistency again occurs when D_{t+1} is greater than Y_t^{produced}. All Y is D, but all D is not (out of) Y, part of the total expenditures being financed by money creation or dishoarding. But how can we write $Y < D$ in the face of identity $Y=D$?

In reality, we must most carefully avoid both inequalities, $D<Y$, and $Y<D$, or else we end up with this absurd result that $Y \neq C+I$, while Y is equal to $C+I$ by definition. This is why the Robertsonian sequence is a *non sequitur*. Inequality $D_{t+1} \neq Y_t^{\text{produced}}$ must be replaced by equivalence $D_{t+1} \equiv Y_t^{\text{produced}}$.

We need only pursue this line of reasoning one step further in order to reach a proposition which escaped ROBERTSON *but which is also beyond*

the reach of the dominant school. If $D_{t+1} = Y_t^{\text{produced}}$ (an equality which is necessary by the logic of definition $Y=C+I$), *income will necessarily remain stable in time unless we have* $D_{t+1} \neq Y_{t+1}^{\text{produced}}$. Now, since it is not true that the level of national income cannot vary in time, D_{t+1} *cannot be identical to* $Y_{t+1}^{\text{produced}}$. In 1972, ROBERTSON remains a modern authority, because the logical fault in his analysis is entirely endorsed by the school which dominates present teaching in most universities.

The majority school critizises ROBERTSON's theory on the ground that it is an unnecessary complication to introduce "days" into the analysis of income determination. But the real and decisive criticism is much stronger. Lags affect in no way the fundamentals of income determination. But once this is granted it follows that *whether lags are introduced or not, expenditures out of disposable incomes cannot produce any new incomes at all.*

B. THE EXACT MEANING OF IDENTITY $Y=C+I$

1° We write Y_t for income produced in period t.

2° If we assume that demand is lagged on income, D_{t+1} (one-"day" lag) is the demand corresponding to income Y_t.

3° More generally, correspondence

$$D_t = Y_t$$

may be understood logically instead of chronologically. We denote by D_t the total purchases of output produced in period t, even if these purchases totally or partially occur in some other periods. Interpreted in this way, identity $D_t = Y_t$ accords both with Robertsonian and with 'post-Keynesian' analyses.

4° Identity $Y_t = D_t$ has only one correct meaning. The value of the physical output of any period t must necessarily be purchased by consumers and investors. In other words, any income produced (supplied) must necessarily be paid for (demanded). Still more precisely, demand relative to output Y_t cannot be greater or less than the value of that output.

5° If identity $Y_t = (C+I)_t$ is correctly understood, no information can be gained from it about the relative size of incomes which are produced in successive periods of time. We know that

$$Y_t = D_t,$$

and

$$Y_{t+1} = D_{t+1},$$

but we know nothing from identity $Y = C + I$ about the difference

$$Y_{t+1} - Y_t.$$

6° The whole dominant school is founded on equation $Y_{t+1} - Y_t = X$, where X is determined by definition $Y = C + I$, wrongly considered as a condition of equilibrium, that is as an ordinary equation with three unknowns.

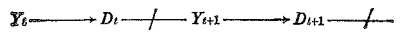
7° As a matter of fact, equation $Y_{t+1} - Y_t = X$ *cannot be solved with the help of identity* $Y = C + I$.

8° The basic mistake of the majority school is clearly identical to ROBERTSON's unwarranted proposition, according to which purchases induce new incomes.

9° Incomes induce purchases, purchases equal to disposable incomes. But purchases induce no new incomes.

10° Identity $Y = C + I$ only means that each and every output must finally be bought and paid for by somebody. From this 'tautology' we can derive no consequence whatever concerning the dynamic variation in national income.

Expenditure D_t corresponds to income Y_t, but it in no way 'belongs' to income Y_{t+1}.



Income Y_t induces expenditures D_t; expenditures D_t do not induce income Y_{t+1}.... The new income, Y_{t+1}, must be explained from some other cause, quite unrelated to identity $Y = C + I$.

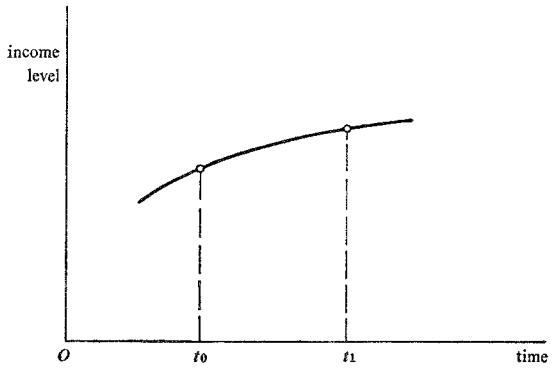
11° This 'other cause' is a purely *virtual factor,* as is explained in the second chapter. Once the size of national income is determined by virtual instead of realized factors, it becomes apparent that ROBERTSON's disquietude about hoarding and dishoarding, and about money creation and destruction, is of no direct relevance to the theory of income determination which rests on an entirely new basis.

12° If we are correct so far, we must be able to find a logical fault at the heart of the dominant theory, a fundamental error which must be directly observable, quite irrespective of ROBERTSON's analysis.

Let us choose SAMUELSON's account of the elementary principles of income determination.

C. WHY SAMUELSON'S DYNAMIC ANALYSIS OF INCOME DETERMINATION IS LOGICALLY UNTENABLE

1° We measure the level of national income on the y axis and time on the x axis.



The level of income can be defined in terms of wage units.

It may be continuously known in time. This only means that at each 'instant' we know how many people are at work in the economy.

Suppose that the curve showing the level of income as a function of time is such that its derivative can be calculated for each time point. It is then easy to determine the absolute value of national income for any period $t_0 - t_n$. But when t_n coincides with t_0, the value of income produced during period $t_0 - t_n$ is zero, however high the level of national income may be at t_0.

2° This result may be expressed in the following way. Two variables can be instantaneously known and a third variable cannot be thus known.

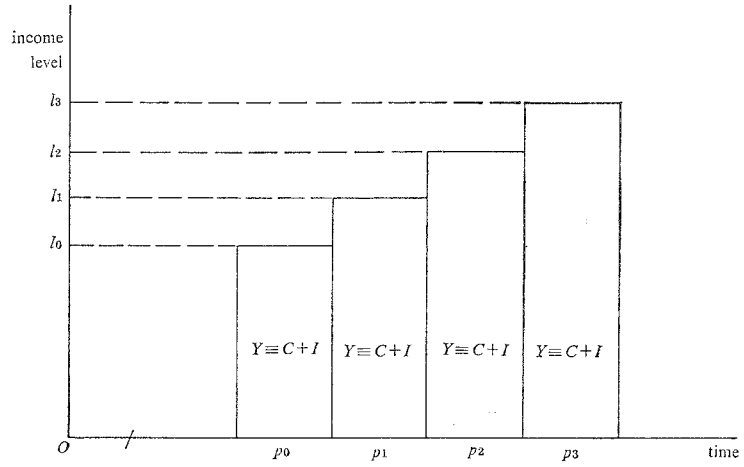
(i) It is possible to determine the instantaneous level of national income.

(ii) It is possible to determine the instantaneous variation in the level of national income.

(iii) It is impossible to determine the instantaneous absolute value of national income.

The phrase 'instantaneous value of national income' is really meaningless. Income can only be measured in a finite period of time.

3° Now, since we must posit a finite time interval—say, a day,—income produced during p_0 takes up the whole period p_0. In other words, it takes a whole day to produce a day's income. Incomes Y_{p0}, Y_{p1}, Y_{p2},.., are produced from the beginning to the end of periods p_1, p_2, p_3,.... This result holds even if we do away with the assumption of a continuous curve.



The *average* level of income produced during period p_0 is l_0. Likewise, l_1, l_2, l_3 are the levels of income produced during p_1, p_2 and p_3. Income Y_{pn} is realized from the beginning to the end of period p_n.

4° We arrive at an undeniable result.

Income Y_{p0} is realized during the whole period p_0.

Since, by definition, $Y=C+I$ for all realized incomes, Y_{p0} is identical to $(C+I)_{p0}$ during the whole period p_0.

We must thus infer that identity $Y=C+I$ obtains for any period of real time of whatever length.

No moment or instant of time can ever be found where identity $Y=C+I$ would not hold.

5° The logical implication of the preceding paragraph is straightforward. *The dynamic level of national income cannot be explained by any adjustment between total supply (Y) and total demand $(C+I)$, for Y is constantly identical to $C+I$.*

6° Two identical factors simply cannot adjust to each other.

If we merely found that Y was constantly equal to $C+I$, the level of national income could still be accounted for by the constant and instantaneous adjustment of Y to $C+I$.

But Y is not merely constantly equal to $C+I$. Income Y is constantly *identical* to demand $C+I$. If we still insisted on explaining the variations in the level of national income by adjusting Y to $C+I$, we would fall into the same absurdity as if our causative factor were the adjustment of Y to Y or of $C+I$ to $C+I$. For identity $Y=C+I$ is just as absolute and unavoidable as identities $Y=Y$ or $C+I=C+I$.

7° Professor Paul A. SAMUELSON bases his macroeconomic theory on identity $Y=C+I$. He should therefore totally resist any temptation to adjust the value of income (Y) to the value of demand (D). But he does not: he simultaneously writes $Y=C+I$ as an identity and as a condition of equilibrium.

"By definition, *national income* (at market prices), Y, can initially be set equal to the sum of consumption expenditure, C, and *net investment*, I:

$$Y=C+I.$$

"If KEYNES had stopped with this identity, we should be left with an indeterminate system. In his simplest model of income determination, he added the following two hypotheses: (*a*) consumption is a function of income, and (*b*) investment may provisionally be taken, at any one time, as a constant. Mathematically, these relations may be written

$$C=C(Y) \text{ and } I=\bar{I}.$$

"When we substitute these into our first identity, we come up with the simplest Keynesian income system:

$$Y=C(Y)+\bar{I}. \tag{1}$$

This is a determinate system, being one equation to determine one unknown variable. While much of the anti-Keynesian and Keynesian world was still arguing over the tautological character of the Keynesian concepts, Professor HANSEN had quickly cut through the non-essentials to isolate the critically important role of the propensity-to-consume schedule, as embodied in this fundamental equation.

"Equation (1) is crucially important for the theory of economic thought. It is the nucleus of the Keynesian reasoning." (*The Collected Scientific Papers*, The M.I.T. Press, 1966, II, pp. 1198–1199)

Some very simple logic quickly shows that the fundamental equation (1)—the nucleus of the Keynesian reasoning—boils down to equation

$$Y = Y.$$

a. It is undeniable that SAMUELSON accepts definition $Y = C + I$. "By definition, *national income* (...)."

b. Realized income is defined by the corresponding realized demand.

c. Income can only be realized in a *finite* time period.

d. Identity $Y = C + I$, which is the definition of national income, is therefore also realized in a *finite* time period.

e. If we consider any series of any successive finite periods, identity $Y = C + I$ holds over the whole time interval covered by this series.

f. Identity $Y = C + I$ is thus verified during any interval of time, however short or long.

g. If consumption is a function of income, we can write $C = C(Y)$.

h. Since $Y = C + I$ for any value of Y (points *e* and *f*),

$$I = Y - C(Y)$$

for any value of Y.

i. So I cannot be a constant, or an exogenous variable, if C is variable as a function of Y.

j. But if net investment is a constant, $I = \bar{I}$, the consumption function is unequivocally known. It must be

$$C = C(Y) = Y - \bar{I},$$

or else Y would not be constantly identical to $C + I$.

k. If we substitute the consumption and the investment functions in definition $Y = C + I$, we simply obtain

$$Y = Y - \bar{I} + \bar{I}.$$

8° SAMUELSON's argument is fallacious only because two conditions are simultaneously satisfied.

(1) $Y = C + I$ defines realized income.

(2) Y is defined in a finite time period.

In the theory of price determination, only one of these two conditions holds.

(1) Supply (s) and demand (d) are defined as identical quantities for any realized price.

(2) $s = d$ is not a definitional identity in any finite period of time but only in an infinitesimal time period.

In price theory, we can adjust supply and demand. No logical error is involved, for identity $s = d$ only holds instantaneously, when transactions actually take place. At any other time, the two factors of demand and supply are separate and they can thus be weighed against each other.

In the theory of income determination, logic does not permit adjustment of Supply and Demand, for identity $Y=C+I$ holds over any time period, however long. There is no conceivable interval of time, however short, where the two factors of Demand and Supply are separable. In brief, Supply and Demand can never be adjusted to each other. The level of national income can never be explained by any adjustment factor between total supply and total demand.

9° SAMUELSON's method is valid in the area of price theory, but it is invalid in the area of income determination. The method fails for the simple reason that, unlike price, income must have a time-dimension.

10° The majority school as a whole teaches two contradictory propositions.

(1) Y is identical to $C+I$.

(2) Y is constantly brought into equality with $C+I$.

Like KEYNES and everybody else, we accept proposition (1). We object to proposition (2).

It is true that proposition (2) could be maintained if Y was only constantly *equal* to $C+I$. But equation $Y=C+I$ is more than a constant equality. It is nothing less than a definitional identity. Now, it seems quite vain to try to bring two identical factors into equality with each other.

11° The level of national income cannot possibly be explained by adjustment between Y and $C+I$, for no process of adjustment is conceivable between the two terms of a constant *identity*.

CHAPTERS II AND III. THE NEW MACROECONOMICS

We shall only mention a few salient points.

VIRTUAL FACTORS

1° Income determination must be explained by an adjustment factor between total demand and total supply.

2° This adjustment factor implies an entirely new definition of effective demand.

3° Effective demand is D^*, where the star denotes *virtual* quantities.

4° No virtual *forces* exist in the area of price theory. Virtual demand is proper to the theory of income determination.

5° Variable D^* (expected Demand in the Keynesian meaning) is entirely distinct from variable D (realized Demand). The two are separate factors, which should never be confounded.

6° Economic laws relative to realized demand, total or partial, cannot be applied to virtual demand, total or partial.

7° The consumption function is $C=C(Y)$ where C is realized and not virtual consumption.

This function cannot be generalized to include virtual consumption. The virtual consumption function is

$$C^*=C^*_0,$$

for only one given (or constant) quantity of consumption is expected by firms producing consumer goods, consumption being measured relatively to the output of a given period.

8° The function of total demand in terms of realized income is

$$D=D(Y)=Y,$$

for realized demand is constantly identical, and therefore constantly equal to realized income.

9° Virtual demand as a function of virtual income must be written

$$D^*=D^*_0,$$

only a *given* quantum of Demand being *expected* (D^*_0) with respect to the output of a given period.

10° The consumption function contributes in no way to the determination of national income. A variation in the propensity to consume has no effect on the size of national income.

11° National income is determined by virtual Demand,

$$Y_t=Y^*_{0_t}=D^*_{0t}.$$

Value $Y^*_{0_t}$ is the equilibrium value of virtual income, for the output of period t. Value Y^*_0 is the only value of income for which it equals demand, when both income and demand are virtual quantities.

12° Once income is determined, it is subdivided into consumption and investment expenditures. This is where the consumption function

comes in. If consumption absorbs a greater part of income, less income is left over for net investment. This is all the consumption function finally amounts to.

13° Analysis of income determination ends up in an unexpected proposition. National income is newly created in each time period, say every month. Incomes of consecutive periods are not linked together, for their determining factor is virtual and not real. Incomes do not form a dynamic chain or concatenation. In other words, the level of income cannot be determined dynamically. Successive incomes are like 'grains' added side by side.

THE KEYNESIAN MULTIPLIER IS NECESSARILY EQUAL TO ONE

1° In the area of virtual quantities,

$$C^* = C^*_0,$$

$$I^* = I^*_0,$$

$$Y^*_0 = C^*_0 + I^*_0,$$

or

$$Y^*_0 = k^*(C^*_0 + I^*_0),$$

where k^* is necessarily equal to one.

2° In the area of realized quantities,

$$C = C(Y),$$

if

$$I = I(Y) = Y - C(Y).$$

More generally,

$$Y = C + I$$

for all values of Y, or

$$Y = k(C + I),$$

where k is necessarily equal to one.

3° Keynesian multipliers greater or smaller than one only appear through the confusing mixture of virtual and realized quantities.

Equation

$$Y = C(Y) + \bar{I}$$

involves a logical error for it combines two irreconcilable elements, one of which is true only in the area of virtual quantities, while the other is true only in the area of realized quantities.

(1) Equation $Y=C(Y)+\bar{I}$ is a condition of equilibrium only in the area of virtual quantities.

(2) Function $C=C(R)$ is valid only in the area of realized quantities.

SAY'S LAW AND MONEY

1° Macroeconomics is intimately tied up with the theory of money.

2° In chapter three of this essay we attempt to show that the Keynesian revolution points to the distinction between nominal money and real money.

3° Newly created money is purely nominal. It offers no purchasing power over commodities.

4° When newly created money is shared out to the factors of production (workers), it changes into real money. It now confers on the final income holders the exact power to purchase goods and services produced by all factors paid in money and not in kind.

5° When real money is finally spent in purchases of investment and consumption goods, it loses all its used purchasing power so as to become purely nominal again.

6° The cycle nominal-real-nominal involves all kinds of money, even money which has not newly been created.

7° While banks are empowered to create money, only firms can create real money.

8° Profits are made up of unspent purchasing power, which is transferred from buyer to seller.

9° Money incomes can only be spent once, by their final holders. Once spent, they are completely destroyed.

10° Every new production issues new purchasing power. This is the final meaning of income determination by virtual factors. There can be no link between the incomes created in successive periods. Incomes induce purchases but purchases induce no new incomes.

11° The distinction of nominal and real money is necessary if SAY's law is really to be invalidated.

"Professor SAMUELSON said that a belief in the validity of the law of markets was a necessary badge of membership in the Classical school of political economy, and comprehension of its logical derivation formed

the *pons asinorum* for the budding theorist. Nevertheless, rather than one organic principle, at least three aspects of this law can be discerned. First, and most zealously held, there is a metaphysical formulation which is irrefutable under all conceivable circumstances and hence empirically meaningless. According to this view, supply *is* demand since goods exchange against goods. Overproduction in one sphere necessarily implies underproduction elsewhere, all values being relative. Second, there is the view that purchasing power is indestructible. What is not spent on consumption is automatically invested; indeed, from SMITH on it was held that what the saver did not consume was passed on as consumption to someone else since capital was regarded in part at least as an advance of subsistence to workers. This second point of view has empirical content and is admittedly false. Effective purchasing power, MV, changes constantly in an upward and downward direction. Finally, and most deserving of attention, is the view that general involuntary unemployment is impossible in a world of perfectly flexible prices." (*The Collected Scientific Papers*, p. 1182)

(1) The first formulation is not really metaphysical. It does have empirical content. In fact, most prices are monetary and goods exchange for money, not for goods.

But SAY's law still holds if it means identity Supply (Y) = Demand ($C+I$) or, identically, S (Saving) = I (Investment).

(2) The second formulation is certainly to the point. SAY's law must be valid if purchasing power is not destructible. For then all factor payments are made out of real money. Spending real money is the very definition of purchases. All goods produced are therefore purchased as soon as their factors are paid.

On the other hand, if factor payments are made out of nominal money, it remains uncertain that real money will eventually be spent to purchase the produced goods.

SAMUELSON's conception of 'destructible' purchasing power is far too weak. Quantity equation $MV=PT$ conveys no idea of the destruction of money incomes. Money incomes can only be destroyed on purchases when real money is converted back into nominal money.

(3) The last formulation is the most interesting. Unemployment equilibrium can occur even when wages are fully flexible.

Entrepreneurs expect Demand D^*_{0t} for the output of period t. If for the same period full employment equilibrium is

$$N_t > D^*_{0t},$$

some workers will be left unemployed.

Underemployment equilibrium can only be fully understood *via* the distinction between nominal and real money. If the public—including profit holders—refrain from spending all their available incomes on final purchases, firms will incur a loss which can only be compensated by 'forced' purchases at the firms' expense of the goods that nobody wants to buy. In such a case identity $Y=C+I$ diminishes profits. In order to avoid the loss, firms curtail employment.

"I propose, therefore, to break away from the traditional method of setting out from the total quantity of money irrespective of the purposes on which it is employed, and to start instead—for reasons which will become clear as we proceed—with the flow of the community's earnings or money-income, and with its twofold division: (1) into the parts which have been *earned* by the production of consumption-goods and of investment-goods respectively, and (2) into the parts which are *expended* on consumption-goods and on savings respectively." (*A Treatise on Money*, I, 134)

This is the profound meaning of the Keynesian revolution. Factor payments are met out of a nominal fund and purchases can only be financed by real money. The distinction between nominal money and real money has fundamental repercussions in all fields of macroeconomics.

NOTES

[1] Monetary price, exchange between commodity and money, is at the same time a nominal and a real expression. The level of monetary price is a purely nominal expression, the monetary 'value' of the physical unit of the goods.

[2] The dynamic level of excess demand can always be represented by a segment; but in income theory, excess demand proper can only be represented by a surface, for income must itself be represented by a surface.

[3] The number of hypothetical curves is of course arbitrary.

[4] R. M. SOLOW, "Economic Growth and Residential Housing," in M. D. KETCHUM and L. T. KENDALL, eds., *Readings in Financial Institutions*, (New York, 1965), p. 146. The quotation is taken from Axel LEIJONHUFVUD, *On Keynesian Economics and the Economics of Keynes, A study in monetary theory*, (Oxford University Press, New York, 1968), p. 4.

[5] SAMUELSON, *The Collected Scientific Papers of Paul A. Samuelson* (MIT, Press, 1966), pp. 1198-1199.

[6] Paul A. SAMUELSON, "The Rate of Interest under Ideal Conditions," in *The Collected Scientific Papers...*, p. 197.

[7] Paul A. SAMUELSON, *The Collected Scientific Papers...*, p. 1199.

[8] (McGraw-Hill Book Company, New York, 1951.)

[9] LERNER, *op. cit.*, pp. 68-69.

[10] See *Economics of Employment*, pp. 80-82.

[11] Macmillan, London, 1967.

[12] *The Collected Scientific Papers* ..., p. 1184.

12 bis

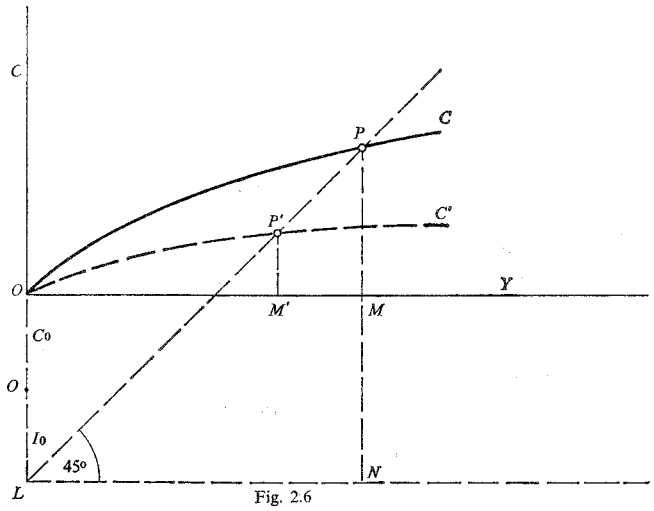


Fig. 2.6

Fig. 15

[13] ROBERTSON, *Essays in Monetary Theory*, (London King and Son, 1940), p. 65.

[14] ROBERTSON, "Alternative Theories of the Rate of Interest," in *The Economic Journal* (1937), p. 429.

[15] KEYNES, "Mr. Robertson on 'Saving and Hoarding'," *The Economic Journal* (1933), p. 700.

[16] ROBERTSON, "Industrial Fluctuation and the Natural Rate of Interest" in *Essays in Monetary Theory*, p. 83.

[17] KEYNES, *loc. cit.*, pp. 699-700.

[18] Not only does he accept KEYNES' definition but he struck upon it before KEYNES. As early as 1931, ROBERTSON criticized KEYNES for excluding business profits from the definition of national income. This allowed the author of *A Treatise on Money* to draw a causative difference between income and the corresponding demand. Total demand includes the spending of profits, but in the *Treatise* profits are banned from the definition of national income. In 1930 KEYNES still attributed a causal importance to the positive or negative difference which can occur between total demand and income thus defined. ROBERTSON showed how profits must on the contrary be included in national income. The result of this inclusion, which KEYNES accepted from 1931 on, and in his *General Theory*, was obviously identity $Y \equiv C + I$, and the logical impossibility of either positive or negative macroeconomic hoarding.
See:
— KEYNES, *A Treatise on Money*, (Macmillan, 1930), I, ch. 9 and 10.
— ROBERTSON, "Mr. Keynes' Theory of Money," *The Economic Journal* (1931), pp. 406-7.
— KEYNES, "A Rejoinder," *The Economic Journal* (1931), p. 422.

[19] KEYNES, *The General Theory*, (Macmillan, 1936), pp. 78-79.

[20] We can shift one curve or the other, or both, even in *static* analysis. But in that case the shift does not involve time. *Dynamic* analysis supposes that we define the new position of the two curves at each time point.

[21] We shall see later that the theoretical consequences of the law of national income determination do not depend on this limitation.

[22] Of course the public keeps its total freedom of option. If factors paid in the second sector spend incomes in the first, it is also true that factors paid in the first sector spend incomes in the second. See below for a complete explanation.

[23] *A Study in Monetary Theory* (Oxford University Press, 1968).

[24] The so-called complete system does not appear in the *General Theory*. It is the work of post-Keynesians, especially of HICKS and HANSEN. In the present study, we need not consider the Hicks-Hansen analysis, since its validity clearly relies on the logical possibility of determining national income from realized consumption and investment.

[25] Joan ROBINSON, "The Theory of Money and the Analysis of Output," *The Review of Economic Studies*, Vol. I, No. 1, (1933).

[26] *Macroeconomic Theory* (Macmillan, New York, 1961), p. 308.

[27] This expression dates back to Sir James STEUART (1767).

[28] Axel LEIJONHUFVUD, *On Keynesian Economics and the Economics of Keynes, A Study in Monetary Theory* (Oxford University Press, 1968), pp. 47-48.

[29] "Readers acquainted with the work of Kuhn will recognize this statement as paraphrasing his description of how 'normal science' prepares the way for 'scientific revolutions'." Cf. T. S. KUHN, *The Structure of Scientific Revolutions*, (Chicago, 1963).

[30] "In the days of KEYNES and PIGOU, a 'monetary economist' was still someone who took the entire economic system as his study object. Today, of course, the label is more likely to designate an economist known for his research on the demand for money, for example. The consumption function and the investment function of the standard model have in the same way developed into specialized fields of study."

[31] If expectations fall short, forced demand is added to spontaneous demand.

[32] *The Collected Scientific Papers of Paul A. Samuelson, op. cit.* "Dynamics, Statics and the Stationary State," p. 201.

[33] Supply is measured on either axis.

[34] A single 'demand quantum' being expected for the production of one given period.

[35] We can therefore write $D^* = f(N^*)$.

[36] Apart from the rate of discount.

[37] "The Cloakroom Rule of International Reserves: Reserve Creation and Resources Transfer," in *International Finance*, ed. Cooper, (Penguin Modern Economics, 1969), p. 337.

[38] The deposit of produced services is more difficult to understand, but the difficulty is not basic.

[39] To both firms, or only to firm I if the selling price of the second firm's output is equal to its cost price.

[40] HANSEN, *A Guide to Keynes*, Economics Handbook Series (McGraw-Hill Publishing Company, London, 1953).

[41] *The Collected...*, "The General Theory," p. 1529.

[42] SAMUELSON, *Foundations of Economic Analysis* (Harvard University Press, Cambridge, 1955). p. 278.

[42 bis] SAMUELSON, *The Collected..., op. cit.*

[43] DON PATINKIN, *Money, Interest, and Prices, An Integration of Monetary and Value Theory*, Second Edition (Harper and Row, New York, 1965), p. 115.

[44] *Theorie der Geld- und Kreditwirtschaft* (Munich, 1914) — The most important chapter of this monograph was translated into English, "Basic Principles of the Money Economy," *International Economic Papers*. No. 9 (1959), pp. 20-38.

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